

Temporal Reconstruction of Hydrogen III: Fine Structure, Lamb Shift, and Radiative Inner-Time Loops

A Self-Contained Post-Freeze DTT Manuscript for Relativistic Corrections, Temporal-Loop Radiative Hierarchies, Source-Firewalled Spectroscopy, and Certificate-Ready Hydrogen Residuals

Mohamed Haj Yousef
Physics Department, United Arab Emirates University
mhajyousef@uaeu.ac.ae

June 2026 – expanded Paper 13 draft

Abstract

This paper is the third hydrogen manuscript in the Dual-Time Theory (DTT) constants-and-hydrogen sequence. Paper 11 formulated hydrogen as an alpha-free neutral proton–electron temporal record resonance. Paper 12 activated the gross Bohr/Rydberg spectrum only after the upstream constants c_* , $\hbar$, α , m_e , and m_p had been derived, frozen, or explicitly calibrated. The present paper treats the next post-freeze layer: fine structure, Lamb shift, recoil, finite-size corrections, and DTT radiative inner-time loops.

The guiding rule is strict: radiative hydrogen formulas may test a previously frozen DTT constants package, but they may not supply a hidden derivation of α , $\hbar$, c_* , the electron mass, or the electron holonomy residual. Earlier hydrogen notes used the electromagnetic inner-time tick

$$\Delta t_r = \alpha \frac{\hbar}{m_e c^2}$$

and asserted that Bohr levels, fine structure, and Lamb-shift scales follow from that single tick. This manuscript preserves the useful temporal-loop vocabulary, but relocates every alpha-containing tick formula to a post-freeze role. In the pre-freeze layer, those formulas are forbidden sources. In the post-freeze layer, they become derived scales and spectroscopy diagnostics.

The manuscript has three technical aims. First, it gives a self-contained correction hierarchy for hydrogen after the gross spectrum has been reconstructed: relativistic kinetic terms, spin-orbit coupling, Darwin/contact terms, the Dirac fine-structure envelope, recoil, hyperfine, finite nuclear size, self-energy, and vacuum polarization. Second, it recasts radiative corrections in DTT language as finite-regulator temporal-loop operators acting on hydrogenic record envelopes. This yields theorem targets for the origin of the order hierarchy

$$\Delta E_{\text{fs}} = O(\mu_H c_*^2 \alpha^4), \quad \Delta E_{\text{Lamb}} = O\left(\frac{\alpha}{\pi} \mu_H c_*^2 \alpha^4 \log \alpha^{-1}\right),$$

without claiming that all QED coefficients have already been derived from first principles. Third, it defines a machine-checkable radiative hydrogen certificate: a frozen input constants package, a baseline QED/Dirac correction ledger, DTT-derived temporal-loop shape columns, no-backflow source firewall, and an overconstraint test forbidding one-amplitude-per-line fitting.

The strongest claim of this paper is conditional. If the upstream DTT constants package is frozen, if the alpha-free hydrogen record geometry descends to the Dirac-Coulomb envelope, and if the temporal-loop operators reproduce the standard radiative coefficient hierarchy without

empirical coefficient matching, then DTT supplies a temporal reconstruction of hydrogen fine structure and Lamb-shift physics. Until that coefficient theorem is discharged, the paper is a rigorous program and certificate architecture, not a completed derivation of all radiative corrections.

Contents

1	Reader guide and proof standard	6
1.1	Three levels of assertion	6
1.2	What this paper deliberately does not claim	7
1.3	Minimal success criterion	7
2	Source firewall and post-freeze status	8
2.1	The alpha-containing tick problem	8
2.2	Role ledger for the source manuscripts	8
2.3	No-backflow theorem	9
3	Post-freeze hydrogen input and notation	9
3.1	The post-freeze input object	9
3.2	State labels and record cycles	10
3.3	Correction decomposition	10
4	Standard correction hierarchy in DTT notation	11
4.1	Why the standard hierarchy must be recovered	11
4.2	Relativistic kinetic, spin-orbit, and Darwin terms	11
4.3	Fine-structure splitting in the $n = 2$ manifold	11
4.4	DTT interpretation of the Dirac envelope	12
5	Temporal-loop radiative architecture	12
5.1	From ordinary loops to temporal loops	12
5.2	Order counting	12
5.3	Temporal ultraviolet and infrared channels	13
6	Fine structure as temporal orientation	14
6.1	Spinorial orientation in the alpha-free datum	14
6.2	Contact and angular channels	14
6.3	Fine-structure correspondence theorem target	15

7	Lamb shift: what must be derived	15
7.1	The Lamb shift as a coefficient problem	15
7.2	Contact-density scaling	15
7.3	A DTT loop decomposition	16
8	Finite inner-time regulator and GUP-like approximations	16
8.1	Why GUP language is useful but dangerous	16
8.2	Effective Hamiltonian from the proxy	17
8.3	Temporal-regulator certificate	17
9	Recoil, finite nuclear size, and hyperfine structure	17
9.1	Why these ledgers are separate	17
9.2	Finite nuclear size	18
9.3	Recoil	18
9.4	Hyperfine structure	18
10	DTT residuals beyond the standard baseline	18
10.1	Residual definition	18
10.2	Allowed shape columns	19
10.3	No line-by-line fitting	19
10.4	Connection to state-dependent holonomy	19
11	Radiative hydrogen certificate	19
11.1	Certificate object	19
11.2	Required fields	20
11.3	Acceptance and failure	20
11.4	Machine-readable schema sketch	20
12	Toy benchmark certificate	21
12.1	Purpose of the benchmark	21
12.2	Toy definitions	21
13	Comparison protocol with spectroscopy	21
13.1	Baseline before residuals	21
13.2	Overconstraint requirement	22
13.3	Recommended channels	22

14 Interface to Paper 14	23
15 Detailed temporal-envelope derivation route	23
15.1 From neutral record geometry to charged spinor envelopes	23
15.2 The spinor-lift theorem target	23
15.3 Foldy-Wouthuysen-like expansion as a certificate check	24
15.4 Required finite-regulator estimates	24
16 Derivation anatomy of the fine-structure terms	24
16.1 Relativistic kinetic correction	24
16.2 Spin-orbit temporal orientation	25
16.3 Darwin/contact term	25
16.4 Fine-structure theorem package	25
17 Radiative-loop renormalization and matching	25
17.1 Why renormalization appears	25
17.2 Three classes of renormalization	25
17.3 Counterterm ledger	26
17.4 Matching theorem target	26
18 Hydrogenic systems beyond ordinary hydrogen	26
18.1 Deuterium and isotope shifts	26
18.2 Muonium	26
18.3 Positronium	27
18.4 Hydrogen-like ions	27
19 Antihydrogen and orientation tests	27
19.1 Charge conjugation	27
19.2 Orientation residual theorem target	27
20 Numerical reporting template	27
21 Proof obligations for a strong Paper 13 claim	28
22 Failure modes and referee stress tests	28
22.1 Failure modes	28
22.2 Referee stress tests	29

23 Detailed fine-structure derivation ledger	29
23.1 Hydrogenic matrix elements used by the envelope	29
23.2 Relativistic kinetic term	30
23.3 Spin-orbit term with temporal orientation	30
23.4 Darwin/contact term	30
23.5 Sum rule to the Dirac envelope	31
24 Radiative coefficient hierarchy and status table	31
24.1 Generic bound-state expansion	31
24.2 Derived vs imported baseline	32
25 Finite-regulator temporal-loop construction	32
25.1 Regulated record cells	32
25.2 Loop weights	32
25.3 Locality classes	33
26 Comparison with older hydrogen tick manuscripts	33
26.1 What is retained	33
26.2 What is repaired	33
26.3 How to cite the older notes in a publication version	34
27 Worked symbolic post-freeze example	34
28 Publication-grade tables to include	34
29 Conclusion	35
A Notation ledger	35
B Claim-status ledger	35
C Figure plan	36
D Validator pseudocode	36
E Minimal bibliography and source roles	37

1 Reader guide and proof standard

One-sentence purpose

Paper 13 explains how hydrogen fine structure and Lamb-shift physics enter the DTT program after the constants are frozen, and it specifies what would be required for a genuine DTT derivation of the radiative coefficients.

This manuscript begins where Paper 12 stops. The gross spectrum has been reconstructed as a downstream consequence of a post-freeze input package,

$$\mathcal{I}_H = (c_\star, \hbar, \alpha, m_e, m_p, \mathcal{D}_H^{\text{af}}, \mathfrak{s}, \mathfrak{u}), \quad (1)$$

where $\mathcal{D}_H^{\text{af}}$ is the alpha-free hydrogen temporal record datum of Paper 11, and $\mathfrak{s}, \mathfrak{u}$ are source-status and unit-dictionary ledgers. Paper 12 then reconstructs

$$a_\mu = \frac{\hbar}{\mu_H c_\star \alpha}, \quad E_n^{(0)} = -\frac{\mu_H c_\star^2 \alpha^2}{2n^2}, \quad R_H = \frac{\mu_H c_\star \alpha^2}{2h}, \quad (2)$$

with $\mu_H = m_e m_p / (m_e + m_p)$. The present paper studies what comes after $E_n^{(0)}$.

The crucial point is that fine structure and Lamb shifts are not independent constants. They are higher-order consequences of the same post-freeze package plus additional structural data: spinorial temporal orientation, relativistic completion geometry, radiative temporal-loop response, nuclear finite-size structure, and recoil. The question is therefore not whether DTT can write the standard formulas. The question is whether DTT can derive the operator hierarchy and coefficients from record geometry without importing them from spectroscopy or QED coefficient matching.

1.1 Three levels of assertion

Level	Meaning	Examples in this paper
Definition	A term or object introduced for later use.	Radiative hydrogen certificate; temporal-loop operator; post-freeze tick; correction ledger.
Conditional theorem	A result proved from stated hypotheses.	If a post-freeze Dirac-Coulomb envelope exists, the standard $O(\alpha^4)$ fine-structure hierarchy follows from its Foldy-type expansion.
Theorem target	A required future derivation.	DTT temporal-loop operators must reproduce the self-energy/vacuum-polarization coefficient hierarchy without fitted radiative constants.
Phenomenology	A comparison procedure after all upstream data are frozen.	Residual-vector tests across hydrogen, deuterium, Rydberg, and antihydrogen transitions.

1.2 What this paper deliberately does not claim

Non-claims

This paper does not claim that α has been derived from hydrogen; that was assigned to the structural-integer, holonomy-fraction, coupling-correspondence, and physical-electron certificate papers. It does not claim that QED has been replaced; it instead organizes the recovery of the standard hydrogen correction hierarchy and identifies the DTT-specific coefficient problem. It does not claim that the Lamb shift is fully derived unless the logarithmic and constant coefficients are obtained from a target-blind temporal-loop certificate.

The older hydrogen notes are valuable because they contain the physical intuition of inner-time loops, finite ticks, GUP-like corrections, and radiative order counting. They are not sufficient as a derivation of α because their tick already contains α . They also should not be read as a completed derivation of all QED radiative coefficients unless the coefficients are reconstructed without importing QED matching data. The present paper keeps the useful intuition and repairs the claim status.

1.3 Minimal success criterion

A complete Paper 13 theorem would require a frozen certificate

$$\mathcal{C}_{H,\text{rad}} = (\mathcal{I}_H, \mathcal{D}_H^{\text{af}}, H_D, \mathcal{O}_{\text{fs}}, \mathcal{L}_{\text{loop}}, \mathcal{R}_{\text{coeff}}, \mathcal{S}_{\text{residual}}, \mathbf{q}, h_{H,\text{rad}}), \quad (3)$$

where H_D is the post-freeze Dirac-Coulomb envelope, $\mathcal{O}_{\text{fs}}$ is the finite-structure operator ledger, $\mathcal{L}_{\text{loop}}$ is the temporal-loop radiative operator family, $\mathcal{R}_{\text{coeff}}$ is the coefficient-discharge ledger, $\mathcal{S}_{\text{residual}}$ is the spectroscopy residual design matrix, $\mathbf{q}$ is the source-firewall audit, and $h_{H,\text{rad}}$ is the target-blind hash.

Criterion 1.1 (Paper 13 strong success criterion). *The strong Paper 13 claim is discharged only if the certificate $\mathcal{C}_{H,\text{rad}}$ derives the fine-structure and Lamb-shift coefficient hierarchy from post-freeze DTT data, uses no pre-freeze hydrogen spectroscopy to tune coefficients, agrees with standard radiative hydrogen corrections within the declared truncation error, and provides at least one overconstrained residual prediction not equivalent to line-by-line fitting.*

Figure placeholder: Position of Paper 13 in the hydrogen sequence

Paper 11 supplies alpha-free temporal hydrogen geometry. Paper 12 activates the Bohr/Rydberg gross spectrum after upstream constants are frozen. Paper 13 adds relativistic and radiative corrections. Paper 14 turns state-dependent residuals into overconstrained spectroscopy tests.

Label: p13-series-position

Figure 1: Position of Paper 13 in the hydrogen sequence. Paper 11 supplies alpha-free temporal hydrogen geometry. Paper 12 activates the Bohr/Rydberg gross spectrum after upstream constants are frozen. Paper 13 adds relativistic and radiative corrections. Paper 14 turns state-dependent residuals into overconstrained spectroscopy tests.

2 Source firewall and post-freeze status

2.1 The alpha-containing tick problem

Earlier hydrogen drafts use the expression

$$\Delta t_r = \alpha \frac{\hbar}{m_e c^2}. \quad (4)$$

In DTT language, this expression is naturally interpreted as an electromagnetic sampling scale: an α -fraction of the electron Compton recurrence. But because it contains α , it cannot be used to choose the electron holonomy cycle, connection, branch, low-band dimension, or residual determinant phase.

In this manuscript the same scale is renamed

$$\tau_{\text{em,e}} := \alpha_{\text{frozen}} \frac{\hbar_{\text{frozen}}}{m_{e,\text{frozen}} c_\star^2}, \quad (5)$$

and is allowed only after the upstream tuple

$$(\mathcal{C}_e, h_e, \alpha_{\text{DTT}}, \hbar_{\text{DTT}}, c_\star, m_e, m_p) \quad (6)$$

has been frozen or explicitly labeled as calibrated. This is not a technicality. Without this firewall, the hydrogen Lamb-shift formulas could become a hidden fit to α .

Firewall Warning 2.1 (No pre-freeze use of hydrogen tick formulas). *A construction is not a non-circular DTT derivation of α , Δ_R , or the electron holonomy fraction if it uses (4), Bohr/Rydberg constants, measured hydrogen transition frequencies, Lamb-shift numerics, or fine-structure splittings to set q_{rec} , $\hbar_\star$, the electron record cycle, branch orientation, regulator trajectory, cell weights, low-band dimension, or determinant phase.*

2.2 Role ledger for the source manuscripts

Source layer	Allowed use here	Forbidden use here
Projection metrology and $\hbar$ papers	Explain constants as bridges and action-phase conversion.	Use SI numerical values as dimensionless predictions.
Local completion cone paper	Supply $c_\star$ as a reconstruction speed after unit dictionary.	Infer α from light-speed numerics.
Canonical complexification and 137	Supply upstream integer $N_3(10) = 137$.	Use the integer theorem to fix radiative coefficients.
Holonomy fraction and electron certificate papers	Supply α_{DTT} only after a frozen electron certificate.	Let hydrogen tune the electron certificate.

Alpha-free paper	hydrogen	Supply neutral tempo- ral resonance, phase op- position, and record- cycle vocabulary.	Import Bohr/Rydberg numerics before con- stants are frozen.
Old tick/Lamb-shift notes	hydrogen	Supply post-freeze temporal-loop intuition and comparison targets.	Use $\Delta t_r = \alpha \hbar / (m_e c^2)$ to derive α .

2.3 No-backflow theorem

Theorem 2.2 (No-backflow from radiative hydrogen to upstream constants). *Suppose a DTT constants package $\mathcal{K}$ is frozen with hash $h_{\mathcal{K}}$ before any Paper 13 correction coefficient or hydrogen transition residual is evaluated. Suppose the Paper 13 certificate records $h_{\mathcal{K}}$, all post-freeze radiative inputs, all coefficient-generation rules, and all residual-shape columns in a separate hash $h_{H,\text{rad}}$. Then any subsequent agreement or disagreement with fine-structure or Lamb-shift data cannot alter the upstream values α_{DTT} , $\hbar_{\text{DTT}}$, $c_{\star}$, m_e , or the electron holonomy certificate without changing the frozen hash.*

Proof. The claim is procedural rather than metaphysical. The upstream constants and electron certificate enter Paper 13 only through the serialized object $\mathcal{K}$. The radiative certificate may compute functions of $\mathcal{K}$, but it contains no permission to mutate $\mathcal{K}$. Any attempted change in $\mathcal{K}$ alters its canonical serialization and therefore its hash. Thus radiative hydrogen can falsify or constrain the upstream package, but it cannot silently tune it. $\square$

3 Post-freeze hydrogen input and notation

3.1 The post-freeze input object

Definition 3.1 (Post-freeze hydrogen-radiative input). *The Paper 13 input object is*

$$\mathcal{I}_{H,\text{rad}} = (\mathcal{I}_H, H_C, \mathcal{H}_{\text{env}}, \mathfrak{L}_{\text{labels}}, \mathfrak{u}, \mathfrak{q}), \quad (7)$$

where $\mathcal{I}_H$ is the Paper 12 post-freeze gross-spectrum input, H_C is the Coulomb-envelope Hamiltonian, $\mathcal{H}_{\text{env}}$ is the hydrogenic record-envelope Hilbert space, $\mathfrak{L}_{\text{labels}}$ fixes state labels (n, ℓ, j, F, m_F) , $\mathfrak{u}$ is the unit dictionary, and $\mathfrak{q}$ is the source-firewall ledger.

The envelope Hamiltonian is

$$H_C = \frac{p^2}{2\mu_H} - \frac{\alpha \hbar c_{\star}}{r}, \quad (8)$$

where we use the ordinary hydrogenic normalization for the post-freeze Coulomb envelope. The corresponding unperturbed levels are

$$E_n^{(0)} = -\frac{\mu_H c_{\star}^2 \alpha^2}{2n^2}. \quad (9)$$

3.2 State labels and record cycles

A hydrogenic state is labeled by

$$a = (n, \ell, j, F, m_F; X), \quad (10)$$

where X records isotope, nuclear spin, external-field conditions, and extraction convention. The DTT record version is a closed state cycle

$$C_a = C_{n\ell j F; X}. \quad (11)$$

Paper 13 uses these cycles in two ways. First, it evaluates standard correction operators on the envelope state. Second, it prepares state-dependent residual shapes for Paper 14.

3.3 Correction decomposition

The full transition frequency is written as

$$h\nu_{a \rightarrow b} = E_b - E_a, \quad (12)$$

with

$$E_a = E_{n_a}^{(0)} + \Delta E_a^{\text{Dirac}} + \Delta E_a^{\text{rec}} + \Delta E_a^{\text{rad}} + \Delta E_a^{\text{nuc}} + \Delta E_a^{\text{hfs}} + \Delta E_a^{\text{DTT-res}} + \dots. \quad (13)$$

Here ΔE^{Dirac} includes relativistic kinetic, spin-orbit, and Darwin terms as organized by the Dirac-Coulomb envelope; ΔE^{rec} includes recoil beyond reduced mass; ΔE^{rad} includes self-energy and vacuum polarization; ΔE^{nuc} includes finite size and polarizability; ΔE^{hfs} includes hyperfine structure; and $\Delta E^{\text{DTT-res}}$ is the genuinely new DTT residual, if any, derived from record geometry rather than fit line by line.

Figure placeholder: Hydrogen correction hierarchy

Diagram showing the gross Bohr/Rydberg level as the base, the Dirac fine-structure envelope as the first correction layer, radiative temporal loops as the second layer, nuclear/recoil/hyperfine as additional ledgers, and DTT residual shapes as the final overconstraint layer.

Label: p13-correction-hierarchy

Figure 2: Hydrogen correction hierarchy. Diagram showing the gross Bohr/Rydberg level as the base, the Dirac fine-structure envelope as the first correction layer, radiative temporal loops as the second layer, nuclear/recoil/hyperfine as additional ledgers, and DTT residual shapes as the final overconstraint layer.

4 Standard correction hierarchy in DTT notation

4.1 Why the standard hierarchy must be recovered

Any DTT account of hydrogen must reproduce a well-tested organization of scales. In powers of α , for light hydrogen-like systems,

$$E^{(0)} = O(\mu_H c_\star^2 \alpha^2), \quad (14)$$

$$\Delta E_{\text{fs}} = O(\mu_H c_\star^2 \alpha^4), \quad (15)$$

$$\Delta E_{\text{Lamb}} = O\left(\frac{\alpha}{\pi} \mu_H c_\star^2 \alpha^4 \log \alpha^{-1}\right), \quad (16)$$

$$\Delta E_{\text{hfs}} = O\left(\alpha^4 \frac{\mu_H^3}{m_e m_p} c_\star^2\right) \times \text{magnetic factors}. \quad (17)$$

A DTT-specific formulation may reinterpret these terms, but it cannot ignore their scale hierarchy.

4.2 Relativistic kinetic, spin-orbit, and Darwin terms

The usual Pauli expansion of the Dirac-Coulomb envelope gives an effective correction operator of the form

$$H_{\text{fs}} = -\frac{p^4}{8\mu_H^3 c_\star^2} + \frac{1}{2\mu_H^2 c_\star^2} \frac{1}{r} \frac{dV}{dr} \mathbf{L} \cdot \mathbf{S} + \frac{\hbar^2}{8\mu_H^2 c_\star^2} \nabla^2 V, \quad (18)$$

where

$$V(r) = -\frac{\alpha \hbar c_\star}{r}. \quad (19)$$

The three terms are respectively the relativistic kinetic correction, spin-orbit coupling, and Darwin/contact term.

Proposition 4.1 (Dirac fine-structure envelope). *If the post-freeze hydrogen record envelope descends to the Dirac-Coulomb envelope with reduced mass μ_H , then to order α^4 the energy shift depends only on n and j :*

$$\Delta E_{nj}^{\text{Dirac}} = -\frac{\mu_H c_\star^2 \alpha^4}{2n^3} \left(\frac{1}{j+1/2} - \frac{3}{4n} \right) + O(\mu_H c_\star^2 \alpha^6). \quad (20)$$

Proof sketch. This is the standard small- α expansion of the Dirac-Coulomb spectrum after subtracting the rest energy and the gross Bohr term. In DTT terms, this proposition is conditional on the descent of the alpha-free temporal resonance to the post-freeze Dirac-Coulomb envelope. The derivation uses only the post-freeze constants and therefore does not feed back into the α certificate. $\square$

4.3 Fine-structure splitting in the $n = 2$ manifold

For $n = 2$, the envelope predicts

$$E_{2P_{3/2}} - E_{2P_{1/2}} = \frac{\mu_H c_\star^2 \alpha^4}{32} + O(\mu_H c_\star^2 \alpha^6). \quad (21)$$

This scale is not a new DTT parameter. It is the $O(\alpha^4)$ consequence of the same post-freeze package that produced the Bohr spectrum.

4.4 DTT interpretation of the Dirac envelope

The DTT interpretation is not that spin-orbit coupling is inserted by hand. The target interpretation is:

$$\text{spinorial temporal orientation} + \text{local completion geometry} \implies H_{\text{fs}}. \quad (22)$$

In this paper we use H_{fs} as the post-freeze envelope correction and state the DTT derivation as a target.

Theorem Target 4.2 (Spinorial temporal-orientation descent). *Let $\mathcal{D}_H^{\text{af}}$ be the alpha-free hydrogen temporal datum of Paper 11, carrying proton/electron phase opposition and a compatible spinorial orientation bundle. After activation of the frozen charge character and Coulomb envelope, the projected spinorial connection should descend to (18) through order α^4 , with no transition-frequency input.*

5 Temporal-loop radiative architecture

5.1 From ordinary loops to temporal loops

In QED, the Lamb shift is organized by radiative loops: electron self-energy, vacuum polarization, vertex corrections, and higher-order terms. In DTT, the corresponding structural idea is that a bound electron record cycle is not a single smooth orbit. It is a stable outer record supported by inner-time recurrences, partial re-creations, and compact phase loops. Radiative corrections are then represented by temporal-loop operators acting on the hydrogenic record envelope.

Definition 5.1 (Temporal-loop operator family). *A post-freeze temporal-loop family is a finite-regulator collection*

$$\mathcal{L}_{\text{loop},r} = \{L_{a,r}^{(k,q)} : a \in \mathfrak{L}_{\text{labels}}, k, q \in \mathcal{A}_r\}, \quad (23)$$

where a labels hydrogenic states, k counts loop order, q records the local/contact, soft, recoil, or nuclear channel, and each $L_{a,r}^{(k,q)}$ is built from post-freeze record data and the derived electromagnetic sampling scale τ_{em} .

The radiative energy correction is then represented schematically as

$$\Delta E_a^{\text{loop}}(r) = \mu_H c_\star^2 \sum_{k \geq 1} \left(\frac{\alpha}{\pi}\right)^k \sum_{s,t} C_{a;k,s,t}^{\text{DTT}}(r) \alpha^s [\log(\alpha^{-1})]^t. \quad (24)$$

A completed DTT radiative theorem must derive the coefficients $C_{a;k,s,t}^{\text{DTT}}$ from temporal-loop geometry. It is not enough to copy them from QED after the fact.

5.2 Order counting

For S-states the leading self-energy scale is contact-sensitive. The generic form is

$$\Delta E_{nS}^{\text{SE}} = \mu_H c_\star^2 \frac{\alpha}{\pi} \frac{\alpha^4}{n^3} \left[A_{41} \log(\alpha^{-2}) + A_{40}(nS) + O(\alpha \log \alpha^{-1}) \right]. \quad (25)$$

For non-S states the contact part is suppressed and angular factors dominate. Vacuum polarization contributes a different local potential channel, often represented in effective form as

$$\Delta H_{\text{VP}} \sim \frac{\alpha}{\pi} \alpha^4 \mu_H c_\star^2 \mathcal{V}_{\text{VP}}(r; \alpha, \mu_H). \quad (26)$$

DTT temporal-loop order counting must explain three facts:

1. why a radiative loop brings a factor of α/π ,
2. why the bound-state contact scale contributes $\alpha^4 \mu_H c_\star^2$,
3. why logarithms of α^{-1} arise from separated soft and hard temporal scales.

Theorem Target 5.2 (Temporal-loop coefficient theorem). *A completed DTT radiative theorem should construct $\mathcal{L}_{\text{loop},r}$ from finite-regulator temporal record data and prove that, after regulator removal,*

$$C_{a;k,s,t}^{\text{DTT}} = C_{a;k,s,t}^{\text{QED}} \quad (27)$$

for all coefficients claimed as recovered, with no fitting to hydrogen transition data. If only the scaling and logarithmic structure are derived, then the claim must be stated as a scaling theorem rather than a full Lamb-shift derivation.

5.3 Temporal ultraviolet and infrared channels

The old hydrogen notes suggest that high-frequency modes decohere when $\omega\tau_{\text{em}} \gg 1$, while low-frequency modes complete multiple ticks coherently. This is a useful physical picture, but it must be converted into a finite-regulator theorem. A convenient abstract implementation is a temporal response kernel

$$K_\tau(\omega) = 1 - \kappa_2(\omega\tau_{\text{em}})^2 + O((\omega\tau_{\text{em}})^4), \quad |K_\tau(\omega)| \rightarrow 0 \quad (|\omega|\tau_{\text{em}} \rightarrow \infty), \quad (28)$$

with regulator-dependent versions $K_{\tau,r}$. The hard-soft split is then represented by an intermediate scale Λ satisfying

$$\mu_H c_\star^2 \alpha^2 \ll \Lambda \ll \mu_H c_\star^2. \quad (29)$$

A logarithm arises only if the Λ -dependence cancels between channels while leaving a scale ratio such as $\log(\alpha^{-2})$.

Proposition 5.3 (Logarithm from scale separation, conditional form). *Suppose a temporal-loop correction has a soft contribution proportional to $\int_{\omega_{\text{soft}}}^\Lambda d\omega/\omega$ and a hard matching contribution proportional to $\int_\Lambda^{\omega_{\text{hard}}} d\omega/\omega$, with $\omega_{\text{soft}} \sim \mu_H c_\star^2 \alpha^2/\hbar$ and $\omega_{\text{hard}} \sim \mu_H c_\star^2/\hbar$. If regulator dependence cancels at Λ , the remaining logarithm is proportional to $\log(\alpha^{-2})$.*

Proof. The sum of the two logarithmic integrals gives $\log(\omega_{\text{hard}}/\omega_{\text{soft}})$ up to finite constants. With the stated scalings, this is $\log(\alpha^{-2})$ plus mass-ratio and unit-normalization constants absorbed into the finite coefficient ledger. This proves the source of the logarithm under the stated scale-separation hypothesis, not the full numerical constant term. $\square$

Figure placeholder: Temporal-loop radiative correction

Suggested diagram: a bound electron record cycle with an inserted inner-time loop, soft temporal excursions, hard local recurrence, and projection back to the outer record envelope. Mark the post-freeze electromagnetic tick τ_{em} and the hard/soft scale split.

Label: p13-temporal-loop-diagram

Figure 3: Temporal-loop radiative correction. Suggested diagram: a bound electron record cycle with an inserted inner-time loop, soft temporal excursions, hard local recurrence, and projection back to the outer record envelope. Mark the post-freeze electromagnetic tick τ_{em} and the hard/soft scale split.

6 Fine structure as temporal orientation

6.1 Spinorial orientation in the alpha-free datum

Paper 11 defined alpha-free proton/electron phase opposition and neutral compatibility. Paper 13 requires a stronger structure: a spinorial orientation bundle over the hydrogen record cycle. Let

$$\mathcal{S}_{H,r} \rightarrow \mathcal{R}_{H,r} \quad (30)$$

be a finite-regulator spinorial record bundle, with local temporal frame connection Ω_r . The post-freeze fine-structure target is that Ω_r , after Coulomb-envelope activation and regulator removal, gives the effective spin-orbit and Darwin terms.

Definition 6.1 (Fine-structure operator ledger). *The fine-structure ledger is*

$$\mathcal{O}_{\text{fs},r} = (T_r^{\text{kin}}, T_r^{\text{SO}}, T_r^{\text{D}}, \Omega_r, \mathcal{P}_{n\ell j,r}, \mathbf{q}_{\text{fs},r}), \quad (31)$$

where T_r^{kin} , T_r^{SO} , and T_r^{D} are the regulated kinetic, spin-orbit, and Darwin/contact operators, $\mathcal{P}_{n\ell j,r}$ are state projectors, and $\mathbf{q}_{\text{fs},r}$ records source admissibility.

6.2 Contact and angular channels

The Darwin term is contact-sensitive because

$$\nabla^2 \left(-\frac{\alpha \hbar c_\star}{r} \right) = 4\pi \alpha \hbar c_\star \delta^{(3)}(r). \quad (32)$$

Thus it affects S-states strongly. Spin-orbit terms carry angular factors through $\mathbf{L} \cdot \mathbf{S}$, with expectation

$$\langle \mathbf{L} \cdot \mathbf{S} \rangle_{\ell j} = \frac{\hbar^2}{2} \left[j(j+1) - \ell(\ell+1) - \frac{3}{4} \right]. \quad (33)$$

These two channels foreshadow the DTT residual shapes in Paper 14: core-local S-state effects tend to scale like n^{-3} , while angular/spinorial effects carry fixed (ℓ, j) factors.

6.3 Fine-structure correspondence theorem target

Theorem Target 6.2 (Fine-structure temporal-orientation theorem). *From the alpha-free neutral hydrogen datum $\mathcal{D}_H^{\text{af}}$, the post-freeze Coulomb envelope, and a compatible spinorial temporal connection Ω_r , construct $\mathcal{O}_{\text{fs},r}$ such that for each fixed (n, ℓ, j) ,*

$$\lim_{r \rightarrow \infty} \langle \psi_{n\ell j, r} | T_r^{\text{kin}} + T_r^{\text{SO}} + T_r^{\text{D}} | \psi_{n\ell j, r} \rangle = \Delta E_{nj}^{\text{Dirac}} + O(\alpha^6). \quad (34)$$

The construction must not use measured fine-structure intervals or hydrogen spectroscopy to choose Ω_r , projectors, or normalization.

7 Lamb shift: what must be derived

7.1 The Lamb shift as a coefficient problem

The Lamb shift is not merely a scale. Its leading order contains logarithmic and constant terms whose coefficients are physically meaningful. A safe generic statement is

$$\Delta E_{n\ell j}^{\text{Lamb}} = \mu_H c_\star^2 \left(\frac{\alpha}{\pi} \right) \alpha^4 \left[A_{41}(n\ell j) \log(\alpha^{-2}) + A_{40}(n\ell j) \right] + O(\mu_H c_\star^2 \alpha^6 \log \alpha^{-1}). \quad (35)$$

For an S-P Lamb interval, the difference of two such expressions is compared to experiment after recoil, finite-size, and hyperfine conventions are fixed.

The older hydrogen notes write more specific expressions and sometimes state numerical agreement. This paper treats those expressions as post-freeze templates and theorem targets. The coefficient A_{41} may be derivable from scale separation and contact density. The finite constant A_{40} is more demanding: it encodes detailed self-energy structure, Bethe-logarithm-type information, and matching conventions. DTT must derive it or clearly mark it as imported.

7.2 Contact-density scaling

For S-states,

$$|\psi_{nS}(0)|^2 = \frac{1}{\pi a_\mu^3 n^3} = \frac{(\mu_H c_\star \alpha)^3}{\pi \hbar^3 n^3}. \quad (36)$$

This explains why many local radiative and finite-size corrections carry an n^{-3} leading dependence. DTT temporal-loop corrections that are local at the reconstruction core should inherit this shape.

Lemma 7.1 (Core-local shape). *If a post-freeze DTT correction is represented to leading order by a local operator $\lambda_{\text{loc}} \delta^{(3)}(r)$, then its S-state expectation scales as n^{-3} , while it vanishes for $\ell > 0$ in the nonrelativistic envelope approximation.*

Proof. The expectation value is $\lambda_{\text{loc}} |\psi_{nS}(0)|^2$, and (36) gives the n^{-3} scaling. For $\ell > 0$, the hydrogenic radial wavefunction vanishes at the origin, so the delta expectation is zero at this order. $\square$

7.3 A DTT loop decomposition

We decompose the Lamb-shift theorem target into four ledgers:

$$\Delta E_a^{\text{Lamb,DTT}} = \Delta E_a^{\text{soft-temp}} + \Delta E_a^{\text{hard-temp}} + \Delta E_a^{\text{polar-vac}} + \Delta E_a^{\text{residual-temp}}. \quad (37)$$

The first two should reproduce self-energy hard/soft structure, the third should reproduce vacuum polarization, and the fourth is a genuinely new DTT residual if the temporal theory predicts one.

Definition 7.2 (Coefficient-discharge ledger). *The coefficient-discharge ledger is a table assigning each coefficient in (35) one of four statuses:*

- (i) **derived**: obtained from finite-regulator temporal-loop data without QED matching or spectroscopy;
- (ii) **standard-imported**: copied from standard QED for baseline comparison, not claimed as DTT-derived;
- (iii) **calibrated**: fit to data, disallowed in a derivation claim but allowed in exploratory phenomenology if labeled;
- (iv) **open**: not yet computed.

Criterion 7.3 (Lamb-shift derivation criterion). *A DTT manuscript may claim to derive the Lamb shift only for the coefficients marked derived in the coefficient-discharge ledger. A paper that imports A_{40} , Bethe-logarithm-type constants, recoil coefficients, or nuclear parameters may still be useful, but its claim must be stated as a reconstruction or reinterpretation, not a first-principles derivation.*

Figure placeholder: Lamb-shift coefficient-discharge ledger

Suggested figure: split the Lamb shift into soft temporal loop, hard temporal loop, vacuum-polarization temporal response, recoil/nuclear ledgers, and residual DTT shape columns. Each coefficient is marked derived, imported, calibrated, or open.

Label: p13-lamb-ledger

Figure 4: Lamb-shift coefficient-discharge ledger. Suggested figure: split the Lamb shift into soft temporal loop, hard temporal loop, vacuum-polarization temporal response, recoil/nuclear ledgers, and residual DTT shape columns. Each coefficient is marked derived, imported, calibrated, or open.

8 Finite inner-time regulator and GUP-like approximations

8.1 Why GUP language is useful but dangerous

The older hydrogen drafts describe the finite inner-time tick as inducing a generalized uncertainty modification. This is a useful low-energy proxy, but it becomes dangerous if its parameter is set by the measured fine-structure constant before the electron certificate is frozen. In Paper 13 the GUP-like form is treated as an effective expansion of temporal-loop data.

A typical proxy is

$$[x_i, p_j] = i\hbar \left(\delta_{ij} + \beta_{\text{temp}} p^2 \delta_{ij} + \cdots \right), \quad (38)$$

with

$$\beta_{\text{temp}} = \kappa_{\beta} \left(\frac{c_{\star} \tau_{\text{em}}}{\hbar} \right)^2, \quad (39)$$

where κ_{β} must be derived from temporal-regulator geometry or labeled as a model parameter. Since τ_{em} contains α , this proxy is post-freeze only.

8.2 Effective Hamiltonian from the proxy

To first order in β_{temp} , the kinetic energy generates an operator of the schematic form

$$\Delta H_{\text{GUP}} = \kappa_p \frac{\beta_{\text{temp}}}{\mu_H} p^4 + \cdots, \quad (40)$$

with a convention-dependent coefficient κ_p . This can reproduce the scale of some high-momentum/contact-sensitive corrections, but it should not be confused with a full Lamb-shift derivation. A true Lamb-shift theorem must also produce logarithms, spinor structure, vacuum polarization, recoil, and finite constants.

Firewall Warning 8.1 (GUP proxy is not a coefficient theorem). *A GUP-like finite-tick correction may provide a useful low-energy parametrization of temporal-regulator effects. It does not by itself prove the Lamb shift unless the complete coefficient hierarchy, including logarithmic and finite terms, is derived from the same target-blind regulator data.*

8.3 Temporal-regulator certificate

At finite regulator r , the temporal response kernel $K_{\tau,r}$, proxy parameter $\beta_{\text{temp},r}$, and effective operators must be serialized:

$$\mathcal{T}_r = (K_{\tau,r}, \beta_{\text{temp},r}, \Delta H_{\text{GUP},r}, \Delta H_{\text{SE},r}, \Delta H_{\text{VP},r}, \Delta H_{\text{rem},r}). \quad (41)$$

Regulator stability requires

$$|\langle \Delta H_{r+1} - \Delta H_r \rangle_a| \leq \epsilon_r(a), \quad \epsilon_r(a) \rightarrow 0. \quad (42)$$

9 Recoil, finite nuclear size, and hyperfine structure

9.1 Why these ledgers are separate

Precision hydrogen spectroscopy is not a pure electron problem. Reduced mass is already included in Paper 12, but higher-order recoil, finite proton radius, polarizability, nuclear magnetic moment, and hyperfine effects all enter at the precision relevant to Lamb-shift comparisons. DTT must keep these as separate ledgers to avoid attributing nuclear or recoil uncertainties to the electron temporal loop.

9.2 Finite nuclear size

The leading finite-size correction for S-states has the form

$$\Delta E_{nS}^{\text{nuc,size}} = \frac{2}{3} \mu_H^3 c_\star^2 \alpha^4 \frac{r_p^2}{\hbar^2} \frac{1}{n^3} + \dots, \quad (43)$$

up to convention and unit choices. The proton radius r_p is not derived in this paper. If it is fit from hydrogen spectroscopy, that fit must not be mistaken for a DTT derivation of electron radiative structure.

9.3 Recoil

Beyond reduced mass, recoil corrections are organized by powers of m_e/m_p :

$$\Delta E_a^{\text{rec}} = \mu_H c_\star^2 \alpha^4 \left(\frac{m_e}{m_p} \right) R_{a,4} + \mu_H c_\star^2 \alpha^5 \left(\frac{m_e}{m_p} \right) R_{a,5} + \dots. \quad (44)$$

The mass-ratio program is a later DTT paper. Until it is discharged, these factors are post-freeze inputs or calibrated dimensionless ratios.

9.4 Hyperfine structure

Hyperfine structure requires nuclear spin and magnetic moment data. A schematic Fermi term is

$$\Delta E_{nS,F}^{\text{hfs}} = E_F(n) \Xi_{F,I,J} + \dots, \quad (45)$$

where $E_F(n) \propto n^{-3}$ and $\Xi_{F,I,J}$ is the angular factor. DTT may eventually interpret hyperfine structure as coupled proton/electron temporal orientation, but this paper treats it as a separate post-freeze ledger.

Criterion 9.1 (Ledger separation criterion). *A Paper 13 comparison table must report electron radiative, recoil, nuclear finite-size, hyperfine, and DTT-residual contributions in separate columns. Combining them into a single fitted residual weakens the derivation claim.*

10 DTT residuals beyond the standard baseline

10.1 Residual definition

After the standard baseline is evaluated from the frozen constants package, define

$$y_i := \nu_i^{\text{obs}} - \nu_i^{\text{base}}(\mathcal{K}_{\text{frozen}}, r_p, \dots), \quad (46)$$

where ν_i^{base} includes the agreed Dirac, QED, recoil, finite-size, and hyperfine terms at the declared truncation order. A DTT-specific residual model is

$$y_i = \sum_{A=1}^{N_{\text{mech}}} c_A S_A(i) + \varepsilon_i. \quad (47)$$

The columns $S_A(i)$ must be derived from DTT mechanisms before comparison. The number of amplitudes N_{mech} must be the number of independently derived mechanisms, not the number of spectral lines.

10.2 Allowed shape columns

Examples of admissible shape columns include:

$$S_{\text{contact}}(nS) = n^{-3}, \quad (48)$$

$$S_{\text{spin}}(n\ell j) = n^{-3} \sigma_{\ell j}, \quad (49)$$

$$S_{\text{Rydberg}}(n, \ell = n - 1) = o(n^{-3}), \quad (50)$$

$$S_{\text{isotope}}(i) = \text{fixed reduced-mass/nuclear-sensitivity pattern}, \quad (51)$$

$$S_{\text{antiH}}(i) = \text{predeclared orientation-conjugation sign pattern}. \quad (52)$$

These are templates. A strong DTT paper must derive which templates are present.

10.3 No line-by-line fitting

No line-by-line fitting

A model with one amplitude per transition is not a prediction. It is a re-labeling of the data. A DTT residual model must use fewer independently derived columns than measured transitions and must declare all columns before the comparison.

10.4 Connection to state-dependent holonomy

Paper 8 introduced state-dependent record-cycle residuals η_a . Paper 13 uses them only after the standard radiative baseline is fixed:

$$S_i = \eta_b - \eta_a, \quad i : a \rightarrow b. \quad (53)$$

For small shifts, the comparison to transition frequencies can be written schematically as

$$\frac{\delta \nu_i^{\text{DTT}}}{\nu_i} \simeq -K_i \frac{S_i}{N_{\text{eff}}}, \quad (54)$$

where $K_i = \partial \ln \nu_i / \partial \ln \alpha$ is a standard sensitivity coefficient and $N_{\text{eff}} \approx \alpha^{-1}$. This is a comparison protocol, not a replacement for bound-state QED.

11 Radiative hydrogen certificate

11.1 Certificate object

Definition 11.1 (Radiative hydrogen certificate). *A Paper 13 certificate is*

$$\begin{aligned} \mathcal{C}_{H,\text{rad}} = & (\mathcal{K}_{\text{frozen}}, \mathcal{I}_{H,\text{rad}}, \mathcal{L}_{\text{states}}, H_C, H_D, \mathcal{O}_{\text{fs}}, \mathcal{L}_{\text{loop}}, \mathcal{T}_{\text{reg}}, \\ & \mathcal{R}_{\text{coeff}}, \mathcal{R}_{\text{recoil}}, \mathcal{R}_{\text{nuc}}, \mathcal{R}_{\text{hfs}}, \mathcal{S}_{\text{residual}}, \mathbf{q}_{\text{source}}, \mathbf{q}_{\text{no-backflow}}, h_{H,\text{rad}}). \end{aligned} \quad (55)$$

11.2 Required fields

Field	Required content
$\mathcal{K}_{\text{frozen}}$	Hashes and values of $c_\star, \hbar, \alpha, m_e, m_p$, with source status.
H_D	Dirac-Coulomb envelope convention and order through $O(\alpha^4)$.
$\mathcal{O}_{\text{fs}}$	Kinetic, spin-orbit, Darwin/contact operators and state projectors.
$\mathcal{L}_{\text{loop}}$	Temporal-loop operator family, regulator, and coefficient-generation rules.
$\mathcal{R}_{\text{coeff}}$	Derived/imported/calibrated/open status for every coefficient.
$\mathcal{R}_{\text{nuc}}$	Proton radius, nuclear polarizability, isotope, and finite-size status.
$\mathcal{S}_{\text{residual}}$	Fixed residual design matrix for DTT-specific tests.
$\mathbf{q}_{\text{source}}$	Scan proving no pre-freeze alpha, spectroscopy, Lamb-shift, or fine-structure data were used upstream.
$h_{H,\text{rad}}$	Canonical serialization hash.

11.3 Acceptance and failure

Criterion 11.2 (Radiative certificate acceptance). *A radiative hydrogen certificate is accepted as a DTT derivation only for coefficients marked derived. It is accepted as a post-freeze reconstruction if it imports standard QED coefficients but keeps them labeled. It fails the derivation claim if any coefficient marked derived is fit from spectroscopy, if the upstream constants are changed after comparison, or if the residual model has one free amplitude per line.*

11.4 Machine-readable schema sketch

Listing 1: Schematic JSON fields for a Paper 13 radiative hydrogen certificate.

```
{
  "paper": "DTT Paper 13",
  "certificate_type": "postfreeze_hydrogen_radiative",
  "upstream_hashes": {
    "electron_certificate_hash": "...",
    "constants_package_hash": "..."
  },
  "source_status": {
    "alpha": "frozen_upstream_or_calibrated",
    "hydrogen_spectroscopy": "comparison_only",
    "qed_coefficients": "derived_imported_calibrated_or_open"
  },
  "fine_structure": {
    "dirac_envelope_order": "alpha^4",
    "operators": ["kinetic", "spin_orbit", "darwin"]
  },
  "lamb_shift": {
    "coefficient_ledger": [
      {"name": "A41", "status": "derived_or_open"},
      {"name": "A40", "status": "derived_or_imported_or_open"}
    ]
  }
}
```

```

},
"residual_model": {
  "shape_columns_predeclared": true,
  "one_amplitude_per_line": false
},
"validation": {
  "no_backflow": true,
  "hash": "...
}
}

```

12 Toy benchmark certificate

12.1 Purpose of the benchmark

The package accompanying this paper includes a toy certificate. It is not a physical hydrogen calculation. It checks that the validator can recompute a Dirac fine-structure order, a schematic Lamb-shift order, and the no-backflow source firewall from a target-blind JSON file.

12.2 Toy definitions

Let the toy values be

$$\alpha_{\text{toy}} = 0.1, \quad \mu_{\text{toy}} c_{\star}^2 = 1, \quad (56)$$

with the $n = 2$ fine-structure toy splitting

$$\Delta E_{\text{fs}}^{\text{toy}} = \frac{\alpha_{\text{toy}}^4}{32} = 3.125 \times 10^{-6}. \quad (57)$$

A schematic S-state Lamb-order proxy is

$$\Delta E_{\text{Lamb}}^{\text{toy}} = \frac{\alpha_{\text{toy}}}{\pi} \alpha_{\text{toy}}^4 \log(\alpha_{\text{toy}}^{-2}). \quad (58)$$

The validator checks these values, confirms that the coefficient ledger is not mislabeled as physical, and verifies that no forbidden pre-freeze sources are marked as active.

Benchmark status

The toy benchmark validates the certificate machinery only. It is not evidence for the physical Lamb shift, the physical electron certificate, or the fine-structure constant.

13 Comparison protocol with spectroscopy

13.1 Baseline before residuals

For each transition $i : a \rightarrow b$, define

$$\nu_i^{\text{base}} = \nu_i^{(0)} + \nu_i^{\text{Dirac}} + \nu_i^{\text{QED}} + \nu_i^{\text{rec}} + \nu_i^{\text{nuc}} + \nu_i^{\text{hfs}}, \quad (59)$$

where the terms are evaluated from the frozen constants package and any external nuclear inputs are explicitly labeled.

The empirical residual is

$$y_i = \nu_i^{\text{obs}} - \nu_i^{\text{base}}. \quad (60)$$

A DTT residual claim then requires a predeclared design matrix:

$$\mathbf{y} = \mathbf{S}\mathbf{c} + \boldsymbol{\varepsilon}, \quad (61)$$

with covariance Σ . The fit statistic is

$$\chi^2 = (\mathbf{y} - \mathbf{S}\mathbf{c})^T \Sigma^{-1} (\mathbf{y} - \mathbf{S}\mathbf{c}). \quad (62)$$

13.2 Overconstraint requirement

Criterion 13.1 (Overconstraint requirement). *Let N_{lines} be the number of independent transition residuals and N_{amp} the number of DTT amplitudes. A meaningful test requires $N_{\text{lines}} > N_{\text{amp}}$ and independently derived columns in $\mathbf{S}$. If $N_{\text{lines}} \leq N_{\text{amp}}$, the comparison is exploratory and cannot establish predictive success.*

13.3 Recommended channels

The comparison layer should prioritize channels with different dependencies:

- **S-state transitions:** sensitive to contact and Lamb-shift terms.
- **D-state and high- ℓ transitions:** suppress core-local effects and test angular/global loop components.
- **Isotope shifts:** separate reduced-mass, finite-size, and electron-loop effects.
- **Rydberg states:** test high- n , high- ℓ , and global-cycle scaling.
- **Antihydrogen:** test orientation conjugation and branch signs without changing upstream constants.

Figure placeholder: Spectroscopy overconstraint matrix

Suggested matrix: rows are transitions; columns are contact, spin, recoil, nuclear, temporal-loop, high- ℓ suppression, isotope, and antihydrogen orientation shapes. Mark which entries are derived, imported, calibrated, or open.

Label: p13-overconstraint

Figure 5: Spectroscopy overconstraint matrix. Suggested matrix: rows are transitions; columns are contact, spin, recoil, nuclear, temporal-loop, high- ℓ suppression, isotope, and antihydrogen orientation shapes. Mark which entries are derived, imported, calibrated, or open.

14 Interface to Paper 14

Paper 14 should not rederive fine structure or Lamb shifts. It should use the baseline and coefficient ledger from Paper 13, then test state-dependent holonomy residuals. The handoff object is

$$\mathcal{H}_{13 \rightarrow 14} = (\mathcal{C}_{H,\text{rad}}, \nu_i^{\text{base}}, \Sigma, \mathbf{S}_{\text{DTT}}^{\text{allowed}}, \mathbf{q}_{\text{no-fit}}). \quad (63)$$

Paper 14 may add new DTT shape columns only if they are derived from state-cycle holonomy geometry, not from inspection of residuals.

Theorem Target 14.1 (Paper 14 handoff theorem). *The output of Paper 13 should be a reproducible baseline and residual covariance such that Paper 14 can evaluate DTT state-dependent holonomy shapes without re-opening the radiative coefficient ledger.*

15 Detailed temporal-envelope derivation route

15.1 From neutral record geometry to charged spinor envelopes

The alpha-free hydrogen geometry of Paper 11 supplies neutral temporal record data, but it does not by itself supply a charged Coulomb Hamiltonian. The post-freeze step is a representation theorem: after the charged-sector boundary character has been fixed upstream, the neutral proton–electron resonance must admit a low-energy charged envelope. This envelope is the object on which fine structure is computed.

A complete derivation should therefore build the following chain at finite regulator:

$$\mathcal{D}_{H,r}^{\text{af}} \longrightarrow \mathcal{H}_{H,r}^{\text{env}} \longrightarrow \mathcal{D}_{H,r}^{\text{spin}} \longrightarrow H_{0,r} + H_{\text{fs},r} + H_{\text{rad},r}.$$

The first arrow is record-envelope projection; the second is spinor lifting; the third is construction of the post-freeze effective Hamiltonian. Each arrow must be canonical or must carry an explicitly declared matching convention. If an arbitrary choice enters at any arrow, the correction calculation becomes a model family rather than a derivation.

15.2 The spinor-lift theorem target

Theorem Target 15.1 (Spinor-lift theorem target). *For every admissible post-freeze hydrogen input, the stable low-energy envelope of the neutral temporal record pair admits a spinor lift whose generators satisfy the Clifford relations associated with the local completion-cone metric, and whose charged-sector temporal connection reduces to the Coulomb spinor connection in the weak-field, one-proton sector.*

This theorem target is deliberately stronger than saying that the Dirac equation is useful. It says that the spinor structure must be recovered from the record/completion data. The usual Dirac equation then becomes the low-energy envelope of temporal reconstruction, not an independent postulate.

15.3 Foldy-Wouthuysen-like expansion as a certificate check

Once the spinor envelope is obtained, the leading fine-structure terms should arise by a controlled block diagonalization of the positive-energy sector. In ordinary language this resembles a Foldy-Wouthuysen expansion; in DTT language it is a low-band decoupling of fast rest-phase recurrence from slow bound-state record motion.

$$H_{\text{env}} = \beta m_e c_\star^2 + \mathcal{O} + \mathcal{E}, \quad H_{\text{eff}} = m_e c_\star^2 + H_0 + H_{\text{kin}}^{(4)} + H_{\text{SO}} + H_{\text{D}} + O(\zeta^6).$$

A certificate should not merely list the final terms. It should show that the positive-energy projection is stable, that the odd/even split is regulator-consistent, and that the remainder is bounded by the claimed order. This mirrors the low-band discipline used in the physical electron certificate.

15.4 Required finite-regulator estimates

Estimate	Purpose
Spinor-lift stability	Shows that the lifted envelope does not depend on arbitrary basis choices.
Positive-energy gap	Controls separation of rest recurrence from bound-state dynamics.
Odd/even decoupling bound	Controls the block-diagonal expansion through the declared order.
Coulomb-envelope matching error	Bounds the difference between temporal charged envelope and post-freeze Coulomb operator.
Fine-structure remainder	Shows that neglected terms are smaller than the claimed uncertainty budget.

16 Derivation anatomy of the fine-structure terms

16.1 Relativistic kinetic correction

The relativistic kinetic correction is the first visible effect of replacing the nonrelativistic completion-energy expansion by the exact massive reconstruction relation. In a post-freeze envelope one may write schematically

$$H_{\text{kin}}^{(4)} = -\frac{p^4}{8\mu_H^3 c_\star^2}.$$

DTT interprets this not as a separate force, but as the curvature of the massive completion relation when projected into an outer record Hamiltonian. The certificate-level task is to show that the same coefficient appears from the local completion cone and reduced two-body mass convention.

16.2 Spin-orbit temporal orientation

Spin-orbit coupling is the first correction that explicitly sees orientation. In temporal-record language, orbital circulation is a record-cycle orientation and spin is an internal phase orientation. Their coupling is a compatibility cost for transporting spin orientation around the charged proton-electron cycle.

$$H_{SO}^{\text{temp}} = \xi_{SO}(r) \mathbf{L}^{\text{rec}} \cdot \mathbf{S}^{\text{rec}}.$$

A strong DTT result would derive the radial factor and Thomas-precession-like coefficient from temporal transport, not insert them by comparison with known fine-structure intervals.

16.3 Darwin/contact term

The Darwin term is naturally interpreted as unresolved short-scale temporal return structure at the proton/electron contact support. Only S-like states feel the leading contact density. This makes the Darwin term a conceptual bridge between fine structure and Lamb-shift contact effects.

$$H_D^{\text{temp}} = \kappa_D \delta_{\text{rec}}(r).$$

The symbol above is schematic. At finite regulator it should be a cell-supported density operator, not an actual distribution. Continuum/contact notation is allowed only after a regulator-stable limit has been proved.

16.4 Fine-structure theorem package

Theorem Target 16.1 (Fine-structure theorem package). *A complete DTT fine-structure theorem consists of: a spinor-lift theorem, a positive-energy low-band theorem, a temporal Foldy-Wouthuysen expansion, a contact-density limit, and a source scan proving that no measured fine-structure interval was used to choose coefficients.*

17 Radiative-loop renormalization and matching

17.1 Why renormalization appears

Temporal-loop corrections sample both very short inner returns and long bound-state record cycles. The resulting integrals can generate divergent or regulator-dependent pieces unless the record theory specifies how local self-interaction is renormalized. This is not a defect of the program; it is the same structural issue that radiative corrections face in ordinary quantum field theory, translated into record language.

17.2 Three classes of renormalization

1. Standard matching: DTT accepts ordinary bound-state QED as the effective low-energy radiative theory after supplying the constants.

2. Derived record renormalization: DTT derives counterterms from finite-regulator record incidence, temporal-loop measures, and boundary character normalization.
3. Phenomenological matching: DTT fits one or more finite constants to data. This is allowed only if labeled as phenomenology and excluded from parameter-free claims.

17.3 Counterterm ledger

Term	Status	Required documentation
Mass counterterm	external or derived	State whether the electron mass is an external input, a later DTT sector output, or a fitted spectral mass.
Charge/coupling counterterm	forbidden pre-freeze	Must not alter the upstream alpha certificate.
Contact counterterm	derived or matched	Must be fixed before Lamb intervals are compared.
Vacuum-polarization kernel	derived or QED-matched	Requires a source label and uncertainty budget.
DTT residual counterterm	dangerous	Allowed only as a fixed mechanism shared across many lines.

17.4 Matching theorem target

Theorem Target 17.1 (Radiative matching theorem). *The finite-regulator temporal-loop measure and counterterm prescription converge to an effective low-energy radiative Hamiltonian whose matrix elements agree with the standard bound-state radiative expansion through the claimed order, with every finite part either derived from record data or explicitly labeled as an external matching input.*

18 Hydrogenic systems beyond ordinary hydrogen

18.1 Deuterium and isotope shifts

Deuterium changes the reduced mass, nuclear spin, finite-size input, and polarizability corrections. It is therefore an important overconstraint channel: the same electron-sector alpha certificate and the same temporal-loop electron operator must work when nuclear data are changed. A DTT residual that only appears in hydrogen but disappears under isotope comparison requires a nuclear explanation.

18.2 Muonium

Muonium replaces the proton by an antimuon. It removes much of the proton-size uncertainty while preserving a two-body electromagnetic bound state. In the DTT program muonium is a clean test of whether temporal-loop corrections are truly charged-lepton envelope effects rather than proton-structure artifacts.

18.3 Positronium

Positronium is more delicate because both particles have equal mass and annihilation channels. It is still valuable because it tests charge-conjugation symmetry, two-body recoil, and temporal orientation without nuclear structure. A separate positronium certificate should not be treated as a trivial reduced-mass substitution.

18.4 Hydrogen-like ions

Hydrogen-like ions probe higher powers of the parameter $Z\alpha$. They are useful for checking the scaling of fine structure and radiative loops, but they also require careful treatment of nuclear size and strong-field binding effects. In DTT they should be used after the hydrogen certificate architecture is stable.

19 Antihydrogen and orientation tests

19.1 Charge conjugation

The alpha-free hydrogen paper introduced proton/electron phase orientation. The holonomy program also requires an electron branch orientation before determinant comparison. Antihydrogen is the natural arena where these orientation conventions can be tested without changing the absolute electromagnetic coupling.

19.2 Orientation residual theorem target

Theorem Target 19.1 (Antihydrogen orientation target). *Under the DTT charge-conjugation operation that maps hydrogen record orientation to antihydrogen record orientation, all post-freeze correction operators are covariant, while any orientation-odd DTT residual column changes sign according to a predeclared representation rule.*

This target is useful because it distinguishes ordinary CPT-compatible bound-state physics from possible DTT orientation residuals. It must be framed carefully: any claim of hydrogen/antihydrogen splitting beyond known physics is a high-stakes empirical prediction and requires an explicit error budget.

20 Numerical reporting template

A future numerical version of this paper should include tables with the following columns. Values may be symbolic, interval-valued, or numerical, but their source status must be stated.

State	Gross	Fine	Lamb	Other	Source status
1S	computed	computed	computed	finite-size/hfs	post-freeze inputs declared

2S	computed	computed	computed	recoil	Lamb convention de- clared
2P1/2	computed	computed	computed	recoil	centroid convention declared
2P3/2	computed	computed	small	recoil	fine splitting target
nD	computed	computed	suppressed	recoil	high-ell residual test

The comparison table should then use transitions, not only levels:

Transition	Prediction	Residual	Allowed interpretation
1S-2S	post-freeze	data minus base- line	global consistency, not alpha fitting
2S-nS	post-freeze	overconstrained	tests contact/log shape
2S-nD	post-freeze	overconstrained	tests ell dependence
H-D isotope pair	post-freeze	differential	tests nuclear/input separation
H-antiH pair	post-freeze	orientation	tests charge-conjugation rule

21 Proof obligations for a strong Paper 13 claim

The mature version of this manuscript should discharge the following proof obligations.

- P13-1. Construct the spinor lift from alpha-free hydrogen record geometry plus post-freeze charged character.
- P13-2. Prove positive-energy low-band stability for the hydrogen envelope.
- P13-3. Derive the fine-structure operators and bound the expansion remainder.
- P13-4. Construct temporal-loop families at finite regulator.
- P13-5. Derive loop measures and logarithmic coefficients.
- P13-6. Specify renormalization and matching conventions without using measured target intervals.
- P13-7. Freeze the radiative certificate before spectral comparison.
- P13-8. Demonstrate overconstraint across multiple transitions and isotopes.
- P13-9. Show compatibility with the upstream alpha certificate and no-backflow lineage.
- P13-10. Publish validator code and machine-readable certificate data.

22 Failure modes and referee stress tests

22.1 Failure modes

Failure mode	Why it matters
Using hydrogen spec- troscopy to set α	Violates the no-backflow rule and invalidates the fine- structure derivation claim.

Marking QED coefficients as DTT-derived without proof	Overclaims the Lamb-shift theorem.
Fitting one amplitude per transition	Removes predictive content.
Combining nuclear, recoil, and electron-loop terms into one residual	Makes the source of discrepancies unidentifiable.
Changing upstream constants after radiative comparison	Breaks certificate freeze discipline.
Using a GUP proxy as a full Lamb-shift derivation	Confuses scaling intuition with coefficient derivation.
Ignoring regulator stability	Allows finite-cutoff artifacts to masquerade as physics.

22.2 Referee stress tests

A skeptical reader should ask:

1. Which constants are upstream frozen inputs, and which are derived in this paper?
2. Which radiative coefficients are DTT-derived rather than copied from QED?
3. Does the certificate record the status of every coefficient?
4. Are hydrogen data used only after the upstream electron certificate is frozen?
5. Are nuclear and recoil effects separated from electron temporal-loop effects?
6. Does any residual claim have fewer amplitudes than measured lines?
7. Is regulator convergence shown or merely asserted?

23 Detailed fine-structure derivation ledger

23.1 Hydrogenic matrix elements used by the envelope

This section records the ordinary matrix elements that a DTT finite-regulator proof must recover in the post-freeze envelope limit. They are not used to derive upstream constants; they are the target record-envelope integrals. For Coulomb states with reduced mass μ_H ,

$$\left\langle \frac{1}{r} \right\rangle_{n\ell} = \frac{1}{a_\mu n^2}, \quad (64)$$

$$\left\langle \frac{1}{r^2} \right\rangle_{n\ell} = \frac{1}{a_\mu^2 n^3 (\ell + 1/2)}, \quad (65)$$

$$\left\langle \frac{1}{r^3} \right\rangle_{n\ell} = \frac{1}{a_\mu^3 n^3 \ell (\ell + 1/2) (\ell + 1)}, \quad \ell > 0, \quad (66)$$

$$|\psi_{nS}(0)|^2 = \frac{1}{\pi a_\mu^3 n^3}. \quad (67)$$

The spin-orbit operator uses $\langle r^{-3} \rangle$ for $\ell > 0$, while the Darwin/contact operator uses $|\psi_{nS}(0)|^2$. These formulas explain why fine structure carries angular factors and why S-state radiative or

finite-size effects frequently scale as n^{-3} .

Theorem Target 23.1 (Record-envelope matrix-element theorem). *Construct the regulated projected hydrogen envelope states $\psi_{n\ell jm,r}$ from $\mathcal{D}_H^{\text{af}}$ and the frozen Coulomb envelope such that each matrix element in (64) is recovered as a regulator limit, with a declared rate $\epsilon_r \rightarrow 0$. The theorem must use the post-freeze constants only through $\mathcal{I}_H$; it must not use measured hydrogen transition frequencies.*

23.2 Relativistic kinetic term

The kinetic correction is

$$T^{\text{kin}} = -\frac{p^4}{8\mu_H^3 c_\star^2}. \quad (68)$$

Using the Coulomb equation, $p^2 = 2\mu_H(E_n^{(0)} - V)$, the expectation of p^4 can be reduced to moments of V . This gives the well-known contribution

$$\Delta E_{n\ell}^{\text{kin}} = -\frac{\mu_H c_\star^2 \alpha^4}{8n^4} \left(\frac{4n}{\ell + 1/2} - 3 \right). \quad (69)$$

Convention warning

The split into kinetic, spin-orbit, and Darwin pieces is representation-dependent. Only their sum through the Dirac envelope is invariant at this order. A DTT certificate should therefore publish both the chosen representation and the final representation-independent level shifts.

The displayed formula is included as a ledger target rather than as a new physical postulate. In a regulator proof, the same value should arise from the discrete completion operator after the nonrelativistic envelope projection.

23.3 Spin-orbit term with temporal orientation

The spin-orbit term can be written as

$$T^{\text{SO}} = \frac{\alpha \hbar c_\star}{2\mu_H^2 c_\star^2} \frac{\mathbf{L} \cdot \mathbf{S}}{r^3} = \frac{\alpha \hbar}{2\mu_H^2 c_\star} \frac{\mathbf{L} \cdot \mathbf{S}}{r^3}. \quad (70)$$

For fixed n, ℓ, j , its expectation is determined by (33) and $\langle r^{-3} \rangle$. The DTT interpretation is that $\mathbf{L} \cdot \mathbf{S}$ is the record-envelope image of relative rotation between orbital completion flow and spinorial temporal frame.

23.4 Darwin/contact term

The Darwin term is

$$T^{\text{D}} = \frac{\hbar^2}{8\mu_H^2 c_\star^2} \nabla^2 V = \frac{\pi \alpha \hbar^3}{2\mu_H^2 c_\star} \delta^{(3)}(r), \quad (71)$$

up to the same unit conventions as (19). It contributes only to S-states in the leading nonrelativistic envelope. DTT must interpret this term as a local reconstruction-core contribution, not as a spatial point-electron assumption.

23.5 Sum rule to the Dirac envelope

Theorem 23.2 (Fine-structure sum rule, post-freeze envelope form). *Assume the post-freeze hydrogen envelope is the Coulomb envelope with spinorial completion and that $T^{\text{kin}} + T^{\text{SO}} + T^{\text{D}}$ is represented in a common convention. Then the sum of the three first-order perturbation expectations equals (20) through order α^4 .*

Proof. The result is the standard Pauli reduction of the Dirac-Coulomb problem. The kinetic piece supplies the ℓ -dependent relativistic correction, the spin-orbit piece supplies the angular j, ℓ splitting, and the Darwin piece cancels the singular S-state mismatch so the total depends only on n and j . This proof uses the ordinary envelope calculation. The DTT-specific work is to derive the envelope and operator descent from record geometry; that remains [Theorem 6.2](#). $\square$

24 Radiative coefficient hierarchy and status table

24.1 Generic bound-state expansion

A convenient bookkeeping form for hydrogenic radiative effects is

$$\Delta E_a^{\text{rad}} = \mu_H c_*^2 \sum_{k \geq 1} \left(\frac{\alpha}{\pi} \right)^k \sum_{s,t} \alpha^s [\log(\alpha^{-1})]^t A_{a;kst}. \quad (72)$$

The coefficient status table is more important than the notation. Every $A_{a;kst}$ must be marked as derived, imported, calibrated, or open.

Coefficient class	Scale	Current status	DTT	Discharge requirement
Leading self-energy logarithm	$(\alpha/\pi)\alpha^4 \log \alpha^{-1}$	theorem target		derive from temporal hard/soft split and core density.
Leading self-energy finite term	$(\alpha/\pi)\alpha^4$	open/imported unless proved		derive finite part of temporal-loop operator without matching to spectroscopy.
Vacuum polarization leading term	$(\alpha/\pi)\alpha^4$	theorem target		derive projected vacuum response kernel and local potential.
Higher one-loop terms	$(\alpha/\pi)\alpha^5$	open		construct next temporal-loop operators and regulator estimates.
Two-loop radiative terms	$(\alpha/\pi)^2 \alpha^4$	open		define two-loop temporal recurrences and prove coefficient factorization.
Recoil-radiative terms	mass-ratio suppressed	separate ledger		couple temporal-loop sector to proton/electron mass hierarchy.
Nuclear polarizability	nuclear structure	external or later-paper input		derive or import nuclear response; do not absorb into electron loop.

24.2 Derived vs imported baseline

It is acceptable for a post-freeze comparison paper to import standard QED coefficients as a baseline, provided the status is explicit. It is not acceptable to mark imported coefficients as DTT-derived. For a publication sequence, this suggests two versions of Paper 13:

1. a baseline-recovery version, using standard coefficients to test source discipline and residual shape columns;
2. a strong-claim version, deriving selected coefficients from temporal-loop finite-regulator data.

Criterion 24.1 (Coefficient honesty criterion). *A coefficient may be described as DTT-derived only if its numerical value follows from a serialized temporal-loop construction that was fixed before comparison to hydrogen data and before importing the corresponding QED coefficient. Otherwise it must be labeled imported, calibrated, or open.*

25 Finite-regulator temporal-loop construction

25.1 Regulated record cells

Let the hydrogen record complex at regulator r have vertices, edges, and two-cells

$$\mathcal{R}_{H,r} = (V_r, E_r, F_r, \partial_r, \omega_r, J_r, \sim_{\text{obs},r}). \quad (73)$$

A temporal-loop insertion is a marked subcycle $\lambda \subset C_{a,r}$ with a local recurrence weight, a phase winding, and a projection-return map. The finite set of all admissible loop insertions is denoted

$$\Lambda_{a,r}^{(k)} = \{\lambda : \lambda \text{ is an admissible } k\text{-loop insertion on } C_{a,r}\}. \quad (74)$$

25.2 Loop weights

Each insertion carries a weight

$$W_r(\lambda) = A_r(\lambda) P_r(\lambda) K_r(\lambda) O_r(\lambda), \quad (75)$$

where A_r is an amplitude from record action, P_r is a projection-return probability or determinant factor, K_r is a temporal response kernel, and O_r is an orientation/spin factor. The loop correction has the schematic form

$$\Delta E_{a,r}^{(k)} = \sum_{\lambda \in \Lambda_{a,r}^{(k)}} W_r(\lambda) \mathcal{E}_r(\lambda). \quad (76)$$

Theorem Target 25.1 (Finite-regulator loop limit). *For each claimed loop order and state label a , prove that $\Delta E_{a,r}^{(k)}$ has a regulator limit and that the limit has the coefficient status asserted in $\mathcal{R}_{\text{coeff}}$. If two regulators r and r' are connected by an admissible refinement map, prove a drift bound*

$$|\Delta E_{a,r'}^{(k)} - \Delta E_{a,r}^{(k)}| \leq B_a^{(k)} \varepsilon_{r,r'}, \quad \varepsilon_{r,r'} \rightarrow 0. \quad (77)$$

25.3 Locality classes

Temporal loops fall into locality classes:

Class	Expected spectral shape	Interpretation
Core-local	n^{-3} for S-states	contact recurrence near the proton/electron reconstruction core.
Angular/spinorial	$n^{-3}\sigma_{\ell j}$	temporal frame orientation and spin-coupled loops.
Soft/global	logarithmic or weakly angular	long temporal excursions sensitive to bound-state scale.
High- ℓ suppressed	$o(n^{-3})$	absence of core support for circular Rydberg states.
Nuclear-coupled	isotope dependent	proton structure and polarizability ledgers.

26 Comparison with older hydrogen tick manuscripts

26.1 What is retained

The older hydrogen notes contain three ideas worth preserving:

1. hydrogen is a temporal resonance, not merely a spatial orbit;
2. finite inner-time sampling supplies a natural language for radiative loop order;
3. high-frequency temporal response may provide a structural alternative to ad hoc ultraviolet cutoffs.

These ideas are compatible with Paper 13 once they are placed after the upstream constants are frozen.

26.2 What is repaired

The same notes sometimes state that the single tick $\Delta t_r = \alpha \hbar / (m_e c^2)$ derives all hydrogen observables. In the present source architecture this must be repaired. Since Δt_r already contains α , it cannot derive α . It can only be a downstream electromagnetic sampling scale unless an independent electron certificate derives α first.

Proposition 26.1 (Tick repair proposition). *If α is supplied by the frozen electron certificate or by an explicitly labeled calibration, then $\tau_{\text{em}} = \alpha \hbar / (m_e c^2)$ is an admissible post-freeze hydrogen scale. If α is taken from hydrogen spectroscopy and then used to justify the same spectroscopy, the construction is circular.*

Proof. The first case is ordinary downstream use of a frozen input. The second case uses the target phenomenon to set the scale that is then claimed to explain the target phenomenon. This violates the no-backflow theorem and the source-firewall rule. $\square$

26.3 How to cite the older notes in a publication version

The publication version should cite the older notes as conceptual or heuristic predecessors, not as completed derivations of α or the full Lamb-shift coefficient set. The revised sentence should be:

Earlier DTT hydrogen notes proposed that a finite electromagnetic inner-time tick organizes Bohr, fine-structure, and Lamb-shift scales. In the present source-firewalled version, that tick is treated as post-freeze unless derived independently from the electron certificate.

27 Worked symbolic post-freeze example

This section demonstrates the bookkeeping without using physical numerical values. Let α , $\mu_H c_\star^2$, and n be symbolic. The $n = 2$ fine-structure splitting from (21) is

$$\Delta E_{2P}^{\text{fs}} = \frac{\mu_H c_\star^2 \alpha^4}{32}. \quad (78)$$

A leading schematic Lamb interval is

$$\Delta E_{2S-2P}^{\text{Lamb,lead}} = \mu_H c_\star^2 \frac{\alpha^5}{\pi} \left[B_{41} \log(\alpha^{-2}) + B_{40} \right] + O(\alpha^6 \log \alpha^{-1}). \quad (79)$$

The coefficient ledger records whether B_{41} and B_{40} are derived, imported, calibrated, or open. A residual DTT prediction may then add

$$\Delta E_{2S-2P}^{\text{DTTres}} = c_{\text{contact}} \left(\frac{1}{2^3} - \rho_{2P} \right) + c_{\text{spin}} \sigma_{2P} + \cdots, \quad (80)$$

where the amplitudes and shapes must be fixed by record-cycle geometry before comparison.

Why the symbolic example matters

The symbolic example shows the order of operations: freeze constants, compute standard baseline, mark coefficient status, then test residual shapes. It does not assert any physical new residual.

28 Publication-grade tables to include

The final manuscript should include the following tables, populated from a real certificate:

1. Upstream constants and hashes: $c_\star, \hbar, \alpha, m_e, m_p$.
2. Fine-structure operator ledger: kinetic, spin-orbit, Darwin, representation convention, order.
3. Lamb-shift coefficient ledger: each coefficient with status derived/imported/calibrated/open.
4. Nuclear/recoil/hyperfine ledger: all non-electron inputs and uncertainties.
5. Temporal-loop regulator table: r , number of loop cells, drift bound, coefficient estimates.
6. Spectroscopy residual table: transition, baseline, residual, covariance, DTT shape columns.
7. Failure/audit table: source-firewall checks, no-backflow checks, one-amplitude-per-line check.

29 Conclusion

This paper makes hydrogen radiative physics usable in the DTT constants program without weakening the non-circular derivation architecture. The key move is not a new formula; it is a source-status correction. Fine structure and Lamb shifts are post-freeze tests and downstream consequences, not inputs to the derivation of α .

The paper proves or records the following:

- the Dirac fine-structure envelope is a post-freeze consequence of $c_\star, \hbar, \alpha, \mu_H$ and spinorial temporal orientation;
- the leading correction hierarchy is organized by $\alpha^4, (\alpha/\pi)\alpha^4 \log \alpha^{-1}$, recoil, nuclear, and hyperfine ledgers;
- DTT temporal loops can be formulated as finite-regulator operators with derived/imported/calibrated/open coefficient statuses;
- the Lamb shift cannot be claimed as fully DTT-derived until logarithmic and finite coefficients are discharged without QED matching or spectroscopy fitting;
- a radiative hydrogen certificate can enforce no-backflow, coefficient-status, and overconstraint discipline.

The next decisive advance is not another qualitative analogy between inner-time loops and QED loops. It is a coefficient theorem: construct the finite-regulator temporal-loop operators, prove their hard/soft scale separation, derive the logarithmic and finite coefficients, and validate the result without tuning to hydrogen lines. Once that is done, Paper 14 can turn hydrogen, deuterium, Rydberg states, and antihydrogen into overconstrained tests of the DTT holonomy-residual program.

A Notation ledger

Symbol	Meaning
$c_\star$	DTT invariant reconstruction speed.
$\hbar$	Action-phase bridge constant.
α	Post-freeze fine-structure constant, supplied upstream or calibrated.
μ_H	Reduced mass $m_e m_p / (m_e + m_p)$.
$\mathcal{D}_H^{\text{af}}$	Alpha-free hydrogen temporal datum from Paper 11.
τ_{em}	Post-freeze electromagnetic inner-time sampling scale.
H_C	Coulomb-envelope Hamiltonian.
H_D	Dirac-Coulomb envelope.
$\mathcal{O}_{\text{fs}}$	Fine-structure operator ledger.
$\mathcal{L}_{\text{loop}}$	Temporal-loop radiative operator family.
$\mathcal{R}_{\text{coeff}}$	Coefficient-discharge ledger.
$\mathcal{S}_{\text{residual}}$	DTT residual design matrix.

B Claim-status ledger

Item	Status	Comment
Dirac fine-structure expansion	Conditional theorem	Follows from post-freeze Dirac-Coulomb envelope; DTT descent from temporal orientation remains a theorem target.
Lamb logarithm scale	Conditional theorem	Scale separation explains logarithms if temporal hard/soft channels satisfy the stated hypotheses.
Lamb finite constants	Open/theorem target	Must be derived from temporal-loop data or marked imported.
GUP-like proxy	Effective model	Useful for finite-tick intuition but not a full radiative derivation.
Recoil/nuclear/hyperfine terms	Separate ledgers	Must not be absorbed into electron temporal-loop residuals.
DTT residual shapes	Theorem targets	Must be predeclared and overconstrained.

C Figure plan

1. Series position diagram for Papers 11–14.
2. Correction hierarchy stack: Bohr, Dirac, Lamb, recoil, nuclear, hyperfine, DTT residual.
3. Temporal-loop insertion on a bound electron record cycle.
4. Hard/soft scale separation producing $\log \alpha^{-1}$.
5. Fine-structure level diagram for the $n = 2$ manifold.
6. Coefficient-discharge ledger diagram.
7. Spectroscopic overconstraint matrix.
8. Certificate freeze and no-backflow dependency graph.

D Validator pseudocode

Listing 2: Pseudocode for validating a Paper 13 radiative hydrogen certificate.

```

load certificate
assert certificate_type == "postfreeze_hydrogen_radiative"
assert source_status.hydrogen_spectroscopy == "comparison_only"
assert source_status.alpha in ["frozen_upstream", "external_calibration"]
assert residual_model.one_amplitude_per_line == false
for coefficient in lamb_shift.coefficient_ledger:
    assert coefficient.status in ["derived", "standard_imported", "calibrated", "open"]
if physical_derivation_claim:
    assert all claimed coefficients have status == "derived"
recompute toy fine_structure_order if benchmark certificate
recompute toy lamb_order if benchmark certificate
compute canonical hash with hash fields blanked
PASS

```

E Minimal bibliography and source roles

The final publication version should cite standard hydrogen and QED sources for the conventional formulas and the internal DTT source manuscripts for the projection-metrology, $\hbar$, $c_\star$, 137, holonomy-fraction, physical-electron certificate, alpha-free hydrogen, and post-freeze Bohr-spectrum layers. The current draft uses the following role separation:

- Standard QED and atomic physics references support the conventional correction hierarchy.
- DTT projection-metrology and $\hbar$ papers support constants-as-bridges and action-phase conversion.
- DTT holonomy and electron-certificate papers support the source firewall and post-freeze α input.
- Older hydrogen tick/Lamb-shift notes support temporal-loop intuition only after quarantine.

References

- [1] H. A. Bethe, The electromagnetic shift of energy levels, *Physical Review* **72**, 339 (1947).
- [2] P. A. M. Dirac, The quantum theory of the electron, *Proceedings of the Royal Society A* **117**, 610–624 (1928).
- [3] K. Pachucki, Theory of the Lamb shift in hydrogen and light hydrogen-like ions, *Physical Review A* **60**, 3593 (1999).
- [4] M. I. Eides, H. Grotch, and V. A. Shelyuto, Theory of light hydrogenlike atoms, *Physics Reports* **342**, 63–261 (2001).
- [5] M. Haj Yousef, Temporal Reconstruction of Hydrogen I: Alpha-Free Proton–Electron Record Geometry, DTT manuscript, 2026.
- [6] M. Haj Yousef, Temporal Reconstruction of Hydrogen II: Derived Constants and the Bohr Spectrum, DTT manuscript, 2026.
- [7] M. Haj Yousef, The Holonomy Fraction in Duality of Time Theory, source-integrated finite-regulator manuscript, 2026.
- [8] M. Haj Yousef, The Physical Electron Certificate, DTT manuscript, 2026.