

Temporal Reconstruction of Hydrogen II: Derived Constants and the Bohr Spectrum

A Self-Contained Dual-Time Theory Manuscript for Post-Freeze Hydrogen
Spectroscopy, Bohr/Rydberg Scales, Reduced-Mass Corrections, and
Source-Quarantined Inner-Time Ticks

Mohamed Haj Yousef
Physics Department, United Arab Emirates University
mhajyousef@uaeu.ac.ae

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Abstract

This paper is the second hydrogen manuscript in the Dual-Time Theory (DTT) constants-and-hydrogen sequence. Paper 11 constructed an alpha-free proton–electron temporal record geometry: neutral record complexes, phase-opposition support, temporal Dirac-type record operators, and branch-orientation data that can be defined without inserting the fine-structure constant into the hydrogen model. The present paper moves to the next layer. It asks what hydrogen becomes once the relevant upstream constants have been derived, certified, or explicitly calibrated: the invariant reconstruction speed $c_\star$, the action-phase bridge $\hbar$, the low-energy fine-structure constant $\alpha_{\text{OS}}(0)$, and the electron and proton rest-mass sectors.

The answer is a source-quarantined reconstruction of the gross hydrogen spectrum. We show how the alpha-free neutral temporal resonance of Paper 11 becomes, after a frozen charged-sector coupling certificate, the ordinary Coulomb envelope with reduced mass $\mu_H = m_e m_p / (m_e + m_p)$. In that downstream regime the familiar hydrogen scales appear as consequences, not inputs:

$$a_\mu = \frac{\hbar}{\mu_H c_\star \alpha}, \quad E_n = -\frac{\mu_H c_\star^2 \alpha^2}{2n^2}, \quad R_H = \frac{\mu_H c_\star \alpha^2}{2h}.$$

The electromagnetic inner-time tick used in earlier hydrogen notes,

$$\tau_{\text{em}} = \alpha \frac{\hbar}{m_e c_\star^2},$$

is reinterpreted as a *post-freeze derived scale*: it may be used only after α , $\hbar$, $c_\star$, and m_e have been fixed by upstream DTT modules or by an explicitly labeled external calibration. It is not allowed as an input to the fine-structure constant certificate.

The manuscript is deliberately self-contained and status-disciplined. It proves the conditional Bohr/Rydberg reconstruction from stated post-freeze inputs, states the DTT correspondence theorem targets that remain to be discharged, gives a finite-regulator hydrogen spectrum certificate format, provides a source firewall preventing backflow from spectroscopy into the α derivation, and prepares the next papers on fine structure, Lamb shift, and differential holonomy tests. It does not claim a new derivation of α , m_e , m_p , proton radius, fine structure, hyperfine splitting, or the Lamb shift. It makes hydrogen the first downstream arena in which the constants derived by the earlier DTT papers become visible as a spectrum.

Contents

1	Reader guide and claim-status discipline	6
1.1	Four layers of hydrogen in the DTT sequence	6
1.2	What is proved, what is assumed, and what is postponed	6
1.3	Why this paper must be post-freeze	7
1.4	Notation and conventions	7
2	Series position and upstream input ledger	8
2.1	The post-freeze hydrogen input object	8
2.2	Input-status table	8
2.3	The no-backflow rule	9
2.4	Conditional theorem of the paper	9
3	Projection metrology and the hydrogen unit dictionary	10
3.1	Internal quantities before external units	10
3.2	Natural, atomic, and DTT internal units	10
3.3	Reduced mass and isotope notation	11
4	From alpha-free neutral resonance to a Coulomb envelope	11
4.1	The alpha-free input from Paper 11	11
4.2	Charged-sector activation	11
4.3	Record envelope, not literal orbit	12
5	Temporal phase-locking and the Bohr derivation	13
5.1	Post-freeze phase-locking condition	13
5.2	Force balance as envelope stationarity	13
5.3	Energy from the stationary envelope	14
5.4	Temporal interpretation of the Bohr radius	14
6	Operator-envelope derivation of the Coulomb spectrum	15
6.1	Envelope Hilbert space and Hamiltonian	15
6.2	Separation and radial equation	15
6.3	Quantum numbers as projected temporal labels	16
7	Bohr, Rydberg, and recurrence scales	16

7.1	Reduced Bohr radius	16
7.2	Rydberg energy and Rydberg constant	16
7.3	Transition frequencies	17
7.4	Compton recurrence and electromagnetic sampling tick	17
8	Finite-regulator hydrogen spectrum certificate	18
8.1	Why a certificate is needed even for standard formulas	18
8.2	Certificate fields	18
8.3	Toy benchmark certificate	19
8.4	Validation theorem target	19
9	Correction hierarchy and what is not included	19
9.1	Leading gross spectrum	19
9.2	Relativistic and spin corrections	20
9.3	Lamb shift and radiative temporal loops	20
9.4	Hyperfine splitting, proton structure, and finite nuclear size	20
9.5	Correction ledger	20
10	Hydrogen comparison protocol	21
10.1	Symbolic before numerical	21
10.2	What may be compared at Paper 12 level	21
10.3	Use of older hydrogen tables	21
11	Compatibility with the alpha program	22
11.1	The full dependency chain	22
11.2	Why hydrogen cannot select the electron holonomy branch	22
11.3	Hydrogen as a test, not a tuning knob	23
12	DTT interpretation of the Bohr formulas	23
12.1	The Bohr radius as projected resonance size	23
12.2	Binding energy as phase-lock deficit	23
12.3	Rydberg constant as frequency image	23
12.4	The electromagnetic tick as a sampling scale	24
13	Finite-regulator route from Paper 11 to Paper 12	24
13.1	Regulated neutral datum	24

13.2 Regulator convergence conditions	24
13.3 Finite-cell resonance isolation	24
14 Algorithmic construction protocol	25
15 Worked symbolic examples	25
15.1 Infinite nuclear mass approximation	25
15.2 Finite proton mass	26
15.3 Toy internal-unit example	26
16 Source quarantine and legacy hydrogen formulas	26
16.1 Legacy formulas that are post-freeze only	26
16.2 How to cite older hydrogen drafts safely	27
17 Interfaces to Papers 13 and 14	27
17.1 Preparation for fine structure and Lamb shift	27
17.2 Preparation for differential holonomy tests	27
17.3 Preparation for mass-ratio tests	27
18 Figures and diagrams planned for a publication version	29
19 Envelope wavefunctions and record-density interpretation	29
19.1 Why wavefunctions appear in a record theory	29
19.2 Radial expectation values as projected sizes	29
19.3 Degeneracy and hidden symmetry	29
20 Isotopes, positronium, muonium, and hydrogen-like systems	30
20.1 Why extensions matter	30
20.2 Deuterium and isotope shifts	30
20.3 Positronium	30
20.4 Muonium	31
20.5 Hydrogen-like ions	31
21 Numerical reporting template	31
21.1 Why numbers require labels	31
21.2 Recommended table columns	32
21.3 Precision warning	32

22 Assumption registry and failure modes	32
22.1 Assumptions	32
22.2 Failure modes	33
23 Conclusion	33
A Appendix A: Derivation details for the Bohr formulas	33
B Appendix B: Source-role ledger	34
C Appendix C: Machine-readable schema sketch	34
D Appendix D: Referee stress tests	34
E Appendix E: Notation ledger	35

1 Reader guide and claim-status discipline

One-sentence purpose

This paper reconstructs the gross hydrogen spectrum from upstream constants; it does not use hydrogen spectroscopy to derive those constants.

The hydrogen atom is the natural proving ground for any constants program. It is simple enough to compute, rich enough to test, and precise enough to expose circularity. In the DTT sequence, hydrogen must therefore play two roles at once. It must be a concrete physical arena where $c_\star$, $\hbar$, α , mass, charge, phase, and record stability meet. It must also be protected from becoming a hidden source of the very constants the program is trying to derive.

Paper 11 made this protection explicit by constructing an alpha-free temporal record geometry for the neutral proton–electron pair. The present paper activates the post-freeze layer: once upstream modules provide the constants, hydrogen produces the Bohr radius, Rydberg scale, binding energies, and transition frequencies. The old inner-time tick formula becomes meaningful here, but only here.

1.1 Four layers of hydrogen in the DTT sequence

Layer	Paper role	Allowed content
Alpha-free geometry	Paper 11	Proton/electron temporal phases, neutral compatibility, phase opposition, temporal record cycles, orientation, no Bohr/Rydberg numerics.
Gross-spectrum activation	Paper 12	Once $c_\star, \hbar, \alpha, m_e, m_p$ are fixed, derive a_μ , E_n , R_H , transition frequencies, and the post-freeze electromagnetic tick.
Radiative and relativistic corrections	Paper 13	Fine structure, spin-orbit terms, Darwin/contact terms, Lamb shift, recoil, finite-size and QED/DTT loop hierarchy.
Differential holonomy tests	Paper 14	State-dependent holonomy residuals η_{nljF} , isotope comparison, high-Rydberg suppression, antihydrogen orientation, overconstraint tests.

1.2 What is proved, what is assumed, and what is postponed

Claim status

The conditional derivation of the Bohr/Rydberg formulas from post-freeze inputs is ordinary and robust. The DTT-specific content is the claim that those inputs are supplied by earlier record/projection/certificate modules, and that the hydrogen resonance geometry of Paper 11 provides the record-level interpretation of the Coulomb envelope. The paper does not claim that the physical electron certificate has already produced α , nor that the electron and proton masses have been derived.

Item	Status	Comment
Post-freeze conditional theorem	Proved in this paper	If $c_\star, \hbar, \alpha, m_e, m_p$ are fixed and the neutral pair admits the Coulomb-envelope correspondence, then the standard nonrelativistic hydrogen spectrum follows.
DTT correspondence target	Theorem target	The alpha-free neutral temporal resonance of Paper 11 must be shown to descend to the effective Coulomb Hamiltonian after the charged-sector boundary response is activated.
Fine-structure constant input	Upstream dependency	α must come from Papers 7–10 or be labeled as external calibration. It cannot be inferred from hydrogen spectroscopy in this paper.
Mass input	Upstream or calibrated dependency	m_e and m_p are required. Later papers may derive temporal mass sectors; this paper can accept them as frozen inputs.
Fine structure and Lamb shift	Deferred	Relativistic, radiative, spin, hyperfine, finite-nuclear-size, and proton-radius effects are not derived here.

1.3 Why this paper must be post-freeze

The older hydrogen notes write an inner-time tick as

$$\Delta t_r = \alpha \frac{\hbar}{m_e c^2}. \quad (1)$$

This expression is physically suggestive: it says that electromagnetic hydrogenic processes sample an α -fraction of the electron Compton recurrence scale. However, it already contains α . Therefore it cannot be used inside a derivation of α . In this paper it is renamed

$$\tau_{\text{em,e}} := \alpha_{\text{frozen}} \frac{\hbar_{\text{frozen}}}{m_{e,\text{frozen}} c_\star^2}, \quad (2)$$

and treated as a downstream hydrogen scale.

Hydrogen source firewall

Before the electron holonomy certificate for α is frozen, one may not use $\Delta t_r = \alpha \hbar / (m_e c^2)$, $a_0 = \hbar / (m_e c \alpha)$, $R_\infty = m_e \alpha^2 c / (2\hbar)$, measured transition frequencies, Lamb-shift numerics, fine-structure splittings, Bohr velocity, or any equivalent spectroscopy datum. After freeze, these formulas may be used as downstream consequences and comparison tools.

1.4 Notation and conventions

Unless explicitly stated otherwise, c below denotes the DTT local-completion invariant $c_\star$, written as c in formulas where no confusion can arise. The Planck bridge satisfies $\hbar = 2\pi \hbar$. The low-energy

on-shell fine-structure constant is denoted $\alpha = \alpha_{\text{OS}}(0)$ when no running-scale ambiguity is present. The electron and proton masses are m_e and m_p , and the reduced mass of hydrogen is

$$\mu_H = \frac{m_e m_p}{m_e + m_p} = m_e \left(1 + \frac{m_e}{m_p} \right)^{-1}. \quad (3)$$

For deuterium or other hydrogenic systems, replace m_p by the nuclear mass M_N and $Z = 1$ by the appropriate nuclear charge Z .

Figure placeholder: Role of Paper 12 in the hydrogen subseries

Paper 11 supplies alpha-free temporal neutral-pair geometry. Papers 7–10 supply or target α . Papers 3–6 supply $\hbar$ and $c_\star$. Paper 12 activates the Bohr/Rydberg layer only after those inputs are frozen.

Label: `paper12-role-in-series`

Figure 1: Role of Paper 12 in the hydrogen subseries. Paper 11 supplies alpha-free temporal neutral-pair geometry. Papers 7–10 supply or target α . Papers 3–6 supply $\hbar$ and $c_\star$. Paper 12 activates the Bohr/Rydberg layer only after those inputs are frozen.

2 Series position and upstream input ledger

2.1 The post-freeze hydrogen input object

Definition 2.1 (Post-freeze hydrogen input object). *A post-freeze hydrogen input object is a tuple*

$$\mathcal{I}_H = (c_\star, \hbar, \alpha, m_e, m_p, \mathcal{D}_H^{\text{af}}, \mathfrak{s}, \mathfrak{u}), \quad (4)$$

where $\mathcal{D}_H^{\text{af}}$ is the alpha-free hydrogen record datum of Paper 11, $\mathfrak{s}$ is a source-status map assigning each constant to a derived, certified, or calibrated origin, and $\mathfrak{u}$ is a unit dictionary mapping internal DTT units to any external unit system used for numerical comparison.

The map $\mathfrak{s}$ is not decoration. It is the first line of defense against circularity. If α is supplied by a future frozen electron certificate, that certificate hash should be recorded. If α is supplied by CODATA for a numerical demonstration, the result must be labeled external calibration and cannot be used as evidence that DTT has derived α .

2.2 Input-status table

Input	DTT role	Required source status
$c_\star$	local-completion speed / causal-cone slope	Derived structurally by the completion-cone paper, or calibrated externally for numerical comparison.

$\hbar$	action-phase bridge	Derived structurally by the action-phase bridge theorem; SI value requires unit dictionary.
α	low-energy charged-sector coupling	Derived by the structural 137 plus electron holonomy certificate, or explicitly labeled external calibration.
m_e	electron temporal inertia / rest recurrence scale	Derived by a future mass-sector paper or calibrated externally.
m_p	proton temporal inertia / nuclear mass input	Derived by future mass hierarchy theory or calibrated externally.
$\mathcal{D}_H^{\text{af}}$	alpha-free neutral hydrogen geometry	Provided by Paper 11 without Bohr/Rydberg data.
$\mathbf{u}$	unit dictionary	Must specify whether values are internal DTT units, SI units, natural units, or symbolic.

2.3 The no-backflow rule

Criterion 2.2 (No-backflow criterion). *A Paper 12 result is admissible only if none of its hydrogen spectral outputs is used to modify, select, tune, or validate the pre-freeze α certificate. Formally, if $\mathcal{C}_e$ is the physical electron certificate of Paper 10 and $\mathcal{C}_{H,\text{Bohr}}$ is the hydrogen spectrum certificate defined in this paper, then the dependency graph must be acyclic:*

$$\mathcal{C}_e \longrightarrow \alpha \longrightarrow \mathcal{C}_{H,\text{Bohr}}, \quad \mathcal{C}_{H,\text{Bohr}} \nrightarrow \mathcal{C}_e. \quad (5)$$

A useful slogan is:

Hydrogen may reveal the constants, but in this sequence it may not secretly choose them.

2.4 Conditional theorem of the paper

Theorem 2.3 (Conditional Bohr/Rydberg reconstruction). *Assume that:*

1. *a post-freeze hydrogen input object $\mathcal{I}_H$ is supplied;*
2. *the alpha-free neutral pair datum $\mathcal{D}_H^{\text{af}}$ descends, after charged-sector activation, to the effective Coulomb envelope*

$$H_H^{(0)} = \frac{p^2}{2\mu_H} - \frac{\alpha\hbar c}{r}; \quad (6)$$

3. *the record-envelope sector admits the usual central-potential separation and square-integrable bound-state condition.*

Then the gross hydrogen spectrum is

$$E_n = -\frac{\mu_H c^2 \alpha^2}{2n^2}, \quad n = 1, 2, 3, \dots, \quad (7)$$

with reduced Bohr radius

$$a_\mu = \frac{\hbar}{\mu_H c \alpha}, \quad (8)$$

and transition frequencies

$$\nu_{n \rightarrow m} = \frac{\mu_H c^2 \alpha^2}{2h} \left(\frac{1}{m^2} - \frac{1}{n^2} \right), \quad n > m. \quad (9)$$

Proof. The proof is the standard Coulomb bound-state computation with the DTT source-status ledger attached. A direct Bohr-phase proof is given in [section 5](#), and an operator-envelope proof is given in [section 6](#). The only DTT-specific hypothesis needed for this theorem is [equation \(6\)](#); this paper does not claim that the Coulomb envelope has been derived from $\mathcal{D}_H^{\text{af}}$ without further work. It states it as the matching theorem target of [section 4](#). $\square$

Main remaining DTT proof obligation

The missing DTT-specific theorem is not the algebra leading from $H = p^2/(2\mu) - \alpha\hbar c/r$ to the Balmer spectrum. It is the correspondence theorem showing that the alpha-free neutral temporal resonance plus frozen charged-sector response produces exactly that Coulomb envelope with the same α certified by the electron boundary response.

3 Projection metrology and the hydrogen unit dictionary

3.1 Internal quantities before external units

Projection metrology distinguishes internal structural quantities from laboratory units. The internal theory may have a reference tick $\tau_\star$, reference length $\ell_\star$, and reference action $S_\star$, related in the stable vacuum regime by bridge laws. External numerical values require a unit dictionary. Hydrogen is especially sensitive to this distinction because the same formulas can be written in SI, natural units, atomic units, or DTT internal units.

Definition 3.1 (Hydrogen unit dictionary). *A hydrogen unit dictionary is a map*

$$\mathbf{u}_H : (\ell_\star, \tau_\star, S_\star, M_\star, Q_\star) \longrightarrow (\text{metre, second, joule second, kilogram, coulomb}) \quad (10)$$

plus a declaration of which constants are exact definitions, which are calibrated measurements, and which are DTT-derived dimensionless invariants.

The DTT derivation target is strongest for dimensionless ratios and integer labels. The gross spectrum, however, includes dimensionful energies and lengths. Therefore Paper 12 should be read in two ways:

1. Symbolically, as a theorem that the hydrogen scales are functions of $\alpha, \hbar, c, m_e, m_p$.
2. Numerically, only after a unit dictionary and source-status ledger are supplied.

3.2 Natural, atomic, and DTT internal units

In natural units $\hbar = c = 1$, the gross hydrogen formulas reduce to

$$a_\mu = \frac{1}{\mu_H \alpha}, \quad E_n = -\frac{\mu_H \alpha^2}{2n^2}. \quad (11)$$

In atomic units for the infinite nuclear mass approximation,

$$\hbar = m_e = e = 4\pi\epsilon_0 = 1, \quad E_n = -\frac{1}{2n^2} \text{ hartree.} \quad (12)$$

In DTT internal units one may instead set $\ell_\star = \tau_\star = S_\star = 1$ at the reference protocol level, then report $\hat{\alpha}$, $\hat{m}_e$, and $\hat{m}_p$ as dimensionless inputs relative to the chosen record standards.

No unit trick

Changing units cannot derive α . A unit dictionary can move dimensions between e , $\hbar$, c , and ϵ_0 , but the dimensionless number α remains a physical invariant. Therefore any numerical match to hydrogen spectroscopy obtained by setting α from data is calibration, not a DTT derivation of α .

3.3 Reduced mass and isotope notation

For a hydrogenic nucleus of mass M_N and charge Z , define

$$\mu_N = \frac{m_e M_N}{m_e + M_N}, \quad a_{\mu,Z} = \frac{\hbar}{\mu_N c Z \alpha}. \quad (13)$$

The leading nonrelativistic spectrum is

$$E_n^{(Z,N)} = -\frac{\mu_N c^2 (Z\alpha)^2}{2n^2}. \quad (14)$$

Paper 12 focuses on $Z = 1$, but keeping Z visible makes later deuterium, muonium, positronium, and hydrogen-like ion tests easier to formulate.

4 From alpha-free neutral resonance to a Coulomb envelope

4.1 The alpha-free input from Paper 11

Paper 11 defined an alpha-free hydrogen datum

$$\mathcal{D}_{H,r}^{\text{af}} = (\mathcal{R}_{p,r}, \mathcal{R}_{e,r}, \mathcal{R}_{H,r}, J_r, \Theta_{p,r}, \Theta_{e,r}, \partial_r, \omega_r, \sim_{\text{obs},r}, \chi_r), \quad (15)$$

with neutral compatibility, phase-opposition support, and neutral temporal cycles. No Bohr radius, Rydberg constant, Coulomb speed ratio, measured transition, or alpha-dependent tick appears in $\mathcal{D}_{H,r}^{\text{af}}$.

The purpose of Paper 12 is not to replace this alpha-free datum with a classical orbit. It is to show how the datum, once coupled to the frozen charged-sector response, admits a record-envelope approximation whose spectrum is the usual hydrogen gross spectrum.

4.2 Charged-sector activation

Definition 4.1 (Post-freeze charged-sector activation). *Let $\mathcal{D}_H^{\text{af}}$ be an alpha-free neutral-pair datum. A post-freeze charged-sector activation is a map*

$$\mathbf{A}_\alpha : \mathcal{D}_H^{\text{af}} \times \{\alpha, \hbar, c, m_e, m_p\} \longrightarrow (\mathcal{H}_H^{\text{env}}, H_H^{\text{env}}, \mathcal{Q}_H), \quad (16)$$

where $\mathcal{H}_H^{\text{env}}$ is a record-envelope Hilbert space, H_H^{env} is a self-adjoint effective Hamiltonian, and $\mathcal{Q}_H$ maps alpha-free temporal resonance labels to envelope quantum numbers.

The leading activation target is

$$H_H^{\text{env}} = \frac{p^2}{2\mu_H} - \frac{\alpha\hbar c}{r} + O(\alpha^4\mu_H c^2), \quad (17)$$

where the correction term collects relativistic, spin, radiative, recoil, and finite-size terms deferred to Paper 13.

Theorem Target 4.2 (Neutral-to-Coulomb correspondence). *Starting from the alpha-free finite record datum $\mathcal{D}_{H,r}^{\text{af}}$, the post-freeze charged-sector response of Papers 8–10 and the boundary coupling correspondence of Paper 9 should produce a record-envelope Hamiltonian whose leading central term is $-\alpha\hbar c/r$, with the same α as the frozen low-energy charged-sector coupling.*

Why this theorem target matters

Without [theorem 4.2](#), this paper is merely a source-disciplined restatement of the standard hydrogen spectrum. With it, DTT would explain why the alpha-free proton–electron temporal resonance becomes the Coulomb problem after the charged-sector boundary response is activated.

4.3 Record envelope, not literal orbit

The Bohr formulas are often introduced through circular orbits. DTT should not regress to a naive spatial-orbit ontology. The correct statement is that circular-orbit formulas are a compact semiclassical encoding of the envelope spectrum. In DTT language:

$$\text{temporal neutral cycle} \rightsquigarrow \text{central record envelope} \rightsquigarrow \text{hydrogenic spectrum}. \quad (18)$$

The radius r_n is not the literal position of a small object circling a proton. It is a scale parameter of the projected record envelope.

Figure placeholder: Neutral temporal resonance to Coulomb envelope

The figure should show the alpha-free proton/electron phase-opposition cycle feeding, after a frozen charged-sector response, into an effective Coulomb envelope with reduced mass.

Label: `neutral-to-coulomb`

Figure 2: Neutral temporal resonance to Coulomb envelope. The figure should show the alpha-free proton/electron phase-opposition cycle feeding, after a frozen charged-sector response, into an effective Coulomb envelope with reduced mass.

5 Temporal phase-locking and the Bohr derivation

5.1 Post-freeze phase-locking condition

In the semiclassical envelope approximation, a closed hydrogenic record cycle Γ_n carries a phase

$$\Delta\theta(\Gamma_n) = \frac{1}{\hbar} \oint_{\Gamma_n} p \cdot dq. \quad (19)$$

Stable single-valued projected records require

$$\oint_{\Gamma_n} p \cdot dq = 2\pi n\hbar, \quad n \in \mathbb{Z}_{>0}. \quad (20)$$

For circular envelopes, this reduces to

$$L_n = \mu_H v_n r_n = n\hbar. \quad (21)$$

In DTT terms, n is not merely a spatial orbital label. It is the integer winding/resonance number of a stable projected temporal record cycle after the Coulomb envelope has been activated.

Definition 5.1 (Principal temporal resonance number). *The principal hydrogen resonance number n is the positive integer for which the compact phase accumulated along a leading closed record-envelope cycle is trivial in $U(1)$:*

$$\exp(i\Delta\theta(\Gamma_n)) = 1, \quad \Delta\theta(\Gamma_n) = 2\pi n. \quad (22)$$

5.2 Force balance as envelope stationarity

For the leading Coulomb envelope, the stationary circular approximation satisfies

$$\frac{\mu_H v_n^2}{r_n} = \frac{\alpha \hbar c}{r_n^2}. \quad (23)$$

Combining [equations \(21\) and \(23\)](#) gives

$$r_n = \frac{n^2 \hbar}{\mu_H c \alpha} = n^2 a_\mu, \quad (24)$$

where

$$a_\mu = \frac{\hbar}{\mu_H c \alpha}. \quad (25)$$

The orbital speed parameter is

$$\frac{v_n}{c} = \frac{\alpha}{n}. \quad (26)$$

This velocity relation should also be post-freeze. Using $v/c = \alpha$ for the ground state to infer α from hydrogen before the electron certificate is circular.

5.3 Energy from the stationary envelope

The kinetic and potential energies are

$$K_n = \frac{1}{2}\mu_H v_n^2 = \frac{\alpha\hbar c}{2r_n}, \quad V_n = -\frac{\alpha\hbar c}{r_n}. \quad (27)$$

Thus

$$E_n = K_n + V_n = -\frac{\alpha\hbar c}{2r_n} = -\frac{\mu_H c^2 \alpha^2}{2n^2}. \quad (28)$$

Proposition 5.2 (Post-freeze Bohr scale reconstruction). *Given $\hbar$, c , α , and μ_H , the projected circular record-envelope approximation yields*

$$r_n = n^2 \frac{\hbar}{\mu_H c \alpha}, \quad v_n = \frac{\alpha c}{n}, \quad E_n = -\frac{\mu_H c^2 \alpha^2}{2n^2}. \quad (29)$$

Proof. Equations [equations \(21\)](#) and [\(23\)](#) imply $v_n = \alpha c/n$ and $r_n = n^2 \hbar/(\mu_H c \alpha)$. Substituting into $E = K + V$ gives [equation \(28\)](#). The derivation is conditional on the post-freeze Coulomb envelope and does not use spectroscopy data to determine α . $\square$

5.4 Temporal interpretation of the Bohr radius

The radius a_μ is the spatial scale at which three independent structures balance:

1. the compact phase bridge $\hbar$,
2. the local completion slope c ,
3. the frozen electromagnetic boundary coupling α ,
4. the temporal inertia scale μ_H .

Therefore, in DTT, the Bohr radius is not primitive. It is the record-level projected length scale of a temporal resonance:

$$a_\mu = \frac{\text{phase-action bridge}}{\text{temporal inertia} \times \text{completion speed} \times \text{charged-sector coupling}}. \quad (30)$$

**Figure placeholder: Bohr phase-locking
as a temporal record-cycle condition**

A closed record envelope accumulates phase $2\pi n$. The circular-orbit picture is shown as a semiclassical shadow of a temporal record-cycle resonance.

Label: `bohr-phase-lock`

Figure 3: Bohr phase-locking as a temporal record-cycle condition. A closed record envelope accumulates phase $2\pi n$. The circular-orbit picture is shown as a semiclassical shadow of a temporal record-cycle resonance.

6 Operator-envelope derivation of the Coulomb spectrum

6.1 Envelope Hilbert space and Hamiltonian

The Bohr derivation is intuitive but semiclassical. The operator-envelope derivation gives the same gross spectrum from the self-adjoint Hamiltonian

$$H_H^{(0)} = -\frac{\hbar^2}{2\mu_H} \nabla^2 - \frac{\alpha \hbar c}{r} \quad (31)$$

acting on $L^2(\mathbb{R}^3, d^3x)$, with bound states $\psi \in L^2$ satisfying

$$H_H^{(0)} \psi = E \psi. \quad (32)$$

The use of $\mathbb{R}^3$ here is an envelope approximation. It should eventually be obtained as a continuum limit of finite record complexes, not assumed as primitive. For Paper 12, it is the standard post-freeze comparison layer.

6.2 Separation and radial equation

Write

$$\psi(r, \theta, \phi) = R_{n\ell}(r) Y_{\ell m}(\theta, \phi), \quad u_{n\ell}(r) = r R_{n\ell}(r). \quad (33)$$

Then $u_{n\ell}$ satisfies

$$-\frac{\hbar^2}{2\mu_H} \frac{d^2 u}{dr^2} + \left[\frac{\hbar^2 \ell(\ell+1)}{2\mu_H r^2} - \frac{\alpha \hbar c}{r} \right] u = E u. \quad (34)$$

Square-integrability and regularity at $r = 0$ quantize the energy.

Theorem 6.1 (Gross Coulomb spectrum). *The bound-state spectrum of [equation \(31\)](#) is*

$$E_n = -\frac{\mu_H c^2 \alpha^2}{2n^2}, \quad n = 1, 2, 3, \dots, \quad (35)$$

with orbital labels $\ell = 0, 1, \dots, n-1$ and magnetic labels $m = -\ell, \dots, \ell$. The characteristic radius is $a_\mu = \hbar/(\mu_H c \alpha)$.

Proof sketch. Set $E = -\hbar^2 \kappa^2 / (2\mu_H)$, introduce $\rho = 2\kappa r$, and solve [equation \(34\)](#) by the usual associated Laguerre ansatz. Termination of the radial series gives

$$\frac{\mu_H \alpha \hbar c}{\hbar^2 \kappa} = n, \quad (36)$$

so $\kappa = \mu_H \alpha c / (\hbar n)$, hence $E = -\mu_H c^2 \alpha^2 / (2n^2)$. The allowed ℓ values follow from regularity and polynomial degree. $\square$

6.3 Quantum numbers as projected temporal labels

The standard labels (n, ℓ, m) should be reinterpreted in DTT as record-envelope labels:

Label	Standard role	DTT interpretation
n	principal energy label	winding/resonance number of closed projected temporal cycle.
ℓ	orbital angular momentum	envelope angular decomposition of neutral-pair record geometry.
m	magnetic projection	orientation label relative to an externally selected record axis.
s, j, F	spin/fine/hyperfine labels	deferred to Paper 13; require spin, nuclear, and radiative temporal-loop structure.

Degeneracy warning

The n^2 -fold orbital degeneracy of the Coulomb problem is a leading-envelope symmetry. Fine structure, Lamb shift, hyperfine splitting, Zeeman shifts, Stark shifts, finite nuclear size, and recoil break parts of that degeneracy. Paper 12 stops at the gross spectrum and only prepares those corrections.

7 Bohr, Rydberg, and recurrence scales

7.1 Reduced Bohr radius

For hydrogen with reduced mass μ_H , define

$$a_\mu = \frac{\hbar}{\mu_H c \alpha}. \quad (37)$$

The infinite-nuclear-mass Bohr radius is

$$a_0 = \frac{\hbar}{m_e c \alpha}. \quad (38)$$

They satisfy

$$a_\mu = a_0 \frac{m_e}{\mu_H} = a_0 \left(1 + \frac{m_e}{m_p} \right). \quad (39)$$

7.2 Rydberg energy and Rydberg constant

The reduced-mass Rydberg energy is

$$\text{Ry}_H = \frac{\mu_H c^2 \alpha^2}{2}. \quad (40)$$

The wavenumber form is

$$R_H = \frac{\text{Ry}_H}{hc} = \frac{\mu_H c \alpha^2}{2h}. \quad (41)$$

The infinite-nuclear-mass Rydberg constant is

$$R_\infty = \frac{m_e c \alpha^2}{2h}. \quad (42)$$

Thus

$$R_H = R_\infty \frac{\mu_H}{m_e} = R_\infty \left(1 + \frac{m_e}{m_p}\right)^{-1}. \quad (43)$$

7.3 Transition frequencies

For a transition from n to $m < n$, the emitted photon frequency is

$$h\nu_{n \rightarrow m} = E_n - E_m = \text{Ry}_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right), \quad (44)$$

so

$$\nu_{n \rightarrow m} = c R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right), \quad \frac{1}{\lambda_{n \rightarrow m}} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right). \quad (45)$$

The Lyman, Balmer, Paschen, and higher series correspond to fixed lower level $m = 1, 2, 3, \dots$

7.4 Compton recurrence and electromagnetic sampling tick

The electron Compton angular frequency and period are

$$\omega_{C,e} = \frac{m_e c^2}{\hbar}, \quad T_{C,e} = \frac{2\pi}{\omega_{C,e}} = \frac{h}{m_e c^2}. \quad (46)$$

The reduced Compton time is

$$\bar{T}_{C,e} = \frac{\hbar}{m_e c^2}. \quad (47)$$

The post-freeze electromagnetic sampling tick is defined by

$$\tau_{\text{em},e} = \alpha \bar{T}_{C,e} = \alpha \frac{\hbar}{m_e c^2}. \quad (48)$$

This is the cleaned-up status of the older DTT hydrogen tick. It is not an axiom of Paper 12. It is a derived downstream scale once α , $\hbar$, m_e , and c are fixed.

Proposition 7.1 (Tick hierarchy after freeze). *After freeze, the leading electron temporal scales obey*

$$\tau_{\text{em},e} : \bar{T}_{C,e} : T_{C,e} = \alpha : 1 : 2\pi. \quad (49)$$

The ground-state Bohr speed relation is

$$\frac{v_1}{c} = \alpha, \quad (50)$$

so the same dimensionless coupling controls both the electromagnetic sampling fraction of the reduced Compton recurrence and the leading Coulomb envelope velocity.

Proof. The scale relation follows from [equations \(46\) and \(48\)](#). The velocity relation follows from [equation \(26\)](#) with $n = 1$. Both statements are post-freeze consequences of the same input α . $\square$

Figure placeholder: Derived hydrogen scales after constant freezeA diagram mapping $\hbar, c, \alpha, m_e, m_p$ to $a_\mu, R_H, E_n, \nu_{nm}$, and $\tau_{\text{em},e}$.Label: `derived-scales`Figure 4: Derived hydrogen scales after constant freeze. A diagram mapping $\hbar, c, \alpha, m_e, m_p$ to $a_\mu, R_H, E_n, \nu_{nm}$, and $\tau_{\text{em},e}$.

8 Finite-regulator hydrogen spectrum certificate

8.1 Why a certificate is needed even for standard formulas

The Bohr/Rydberg formulas are standard, but in the DTT program the source-status discipline matters as much as the formulas. A future implementation should ship a hydrogen spectrum certificate declaring which constants were used, whether they are derived/certified/calibrated, how units were chosen, and whether the output is allowed to feed back into any upstream certificate.

Definition 8.1 (Hydrogen gross-spectrum certificate). *A hydrogen gross-spectrum certificate is a tuple*

$$\mathcal{C}_{H,\text{Bohr}} = (\mathcal{I}_H, \mu_H, a_\mu, R_H, \{E_n\}_{n \leq N}, \{\nu_{n \rightarrow m}\}_{n \leq N, m < n}, \mathfrak{q}_{\text{source}}, \mathfrak{q}_{\text{unit}}, \mathfrak{q}_{\text{correction}}, \mathfrak{q}_{\text{no-backflow}}, h_H). \quad (51)$$

Here $\mathfrak{q}_{\text{source}}$ is the source-status ledger, $\mathfrak{q}_{\text{unit}}$ is the unit dictionary, $\mathfrak{q}_{\text{correction}}$ declares which corrections are included or deferred, $\mathfrak{q}_{\text{no-backflow}}$ records the firewall check, and h_H is a canonical serialization hash.

8.2 Certificate fields

Field	Type	Purpose
<code>constants</code>	object	Values or symbolic placeholders for $c, \hbar, \alpha, m_e, m_p$.
<code>source_status</code>	object	Derived/certified/calibrated/external for each input.
<code>unit_dictionary</code>	object	Internal units, SI units, natural units, or symbolic settings.
<code>reduced_mass</code>	number/symbolic	$\mu_H = m_e m_p / (m_e + m_p)$.
<code>bohr_radius</code>	number/symbolic	$a_\mu = \hbar / (\mu_H c \alpha)$.
<code>rydberg_constant</code>	number/symbolic	$R_H = \mu_H c \alpha^2 / (2h)$.
<code>levels</code>	array	E_n for declared n -range.
<code>transitions</code>	array	$\nu_{n \rightarrow m}$ or wavenumbers for declared transitions.
<code>corrections</code>	object	Included/deferred correction hierarchy.
<code>no_backflow</code>	boolean	Must be true for admissibility.

hash	string	Canonical hash of certificate without hash field.
------	--------	---

8.3 Toy benchmark certificate

The package accompanying this paper includes a toy certificate with nonphysical values

$$c = 1, \text{quad}\hbar = 1, \quad \alpha = \frac{1}{10}, \quad m_e = 1, \quad m_p = 100. \quad (52)$$

The validator recomputes μ_H , a_μ , R_H , E_n , and selected transitions, checks the no-backflow flag, and verifies the canonical hash discipline. This benchmark proves only that the certificate machinery is executable; it does not certify a physical hydrogen spectrum.

Listing 1: Schematic certificate fragment. The physical package includes a machine-readable template and toy benchmark.

```
{
  "paper": "DTT Paper 12",
  "claim_type": "post_freeze_gross_spectrum",
  "constants": {
    "c": {"value": 1.0, "status": "toy"},
    "hbar": {"value": 1.0, "status": "toy"},
    "alpha": {"value": 0.1, "status": "toy_post_freeze"},
    "m_e": {"value": 1.0, "status": "toy"},
    "m_p": {"value": 100.0, "status": "toy"}
  },
  "no_backflow": true,
  "levels_requested": [1,2,3,4]
}
```

8.4 Validation theorem target

Theorem Target 8.2 (Hydrogen spectrum certificate reproducibility). *Given a canonical serialized $\mathcal{C}_{H,\text{Bohr}}$, independent implementations should compute the same reduced mass, Bohr radius, Rydberg constant, level list, transition list, correction-status ledger, no-backflow result, and hash.*

This target is easy for the gross spectrum and becomes more demanding in Papers 13 and 14, where radiative, fine-structure, hyperfine, and holonomy-residual corrections enter.

9 Correction hierarchy and what is not included

9.1 Leading gross spectrum

Paper 12 includes only the leading Coulomb gross spectrum with reduced mass. In terms of orders in α , the binding scale is

$$E_{\text{gross}} \sim \mu_H c^2 \alpha^2. \quad (53)$$

The omitted terms are not small conceptually; they are simply next in the sequence.

9.2 Relativistic and spin corrections

Fine structure enters at order

$$\Delta E_{\text{fs}} \sim \mu_H c^2 \alpha^4. \quad (54)$$

It includes relativistic kinetic correction, spin-orbit coupling, and Darwin/contact terms. In DTT these should correspond to refined temporal-envelope orientation, spin record structure, and local contact behavior of the projected electron record. Paper 13 will treat them.

9.3 Lamb shift and radiative temporal loops

The Lamb shift is a radiative correction with a leading structure of the schematic form

$$\Delta E_{\text{Lamb}} \sim \mu_H c^2 \alpha^5 \left(\ln \frac{1}{\alpha} + C \right) + \dots. \quad (55)$$

Older DTT hydrogen notes include Lamb-shift estimates based on finite inner-time ticks and QED-style radiative factors. Paper 12 does not use those estimates as proof. It marks them as future Paper 13 material requiring a regulator-level temporal-loop derivation.

9.4 Hyperfine splitting, proton structure, and finite nuclear size

Hyperfine splitting depends on spin, nuclear magnetic moment, and proton structure. Finite nuclear size depends on the proton charge radius. These are not fixed by $\alpha, \hbar, c, m_e, m_p$ alone. A source-disciplined DTT account must add the appropriate nuclear temporal record data. Paper 12 therefore does not claim to predict hyperfine structure or proton radius effects.

9.5 Correction ledger

Effect	Status in Paper 12	Later requirement
Reduced mass	Included	Requires m_p or nuclear mass input.
Gross Coulomb spectrum	Included	Requires neutral-to-Coulomb correspondence theorem.
Relativistic kinetic correction	Deferred	Paper 13 temporal-envelope expansion.
Spin-orbit and Darwin terms	Deferred	Paper 13 spin/orientation record sector.
Lamb shift	Deferred	Paper 13 radiative temporal-loop derivation.
Hyperfine splitting	Deferred	Nuclear spin and magnetic moment record data.
Finite proton radius	Deferred	Proton internal record geometry.
External fields	Deferred	Zeeman/Stark source-response modules.
Differential holonomy residuals	Deferred	Paper 14 state-dependent holonomy program.

10 Hydrogen comparison protocol

10.1 Symbolic before numerical

A DTT hydrogen comparison should proceed in the following order:

1. Freeze the input object $\mathcal{I}_H$ and record its source statuses.
2. Choose symbolic, natural, internal, or SI units.
3. Generate $\mathcal{C}_{H,\text{Bohr}}$ without using measured transition frequencies to adjust any upstream input.
4. Compute gross-spectrum predictions.
5. Add declared correction layers, if the relevant later papers have discharged them.
6. Compare with spectroscopy only after the certificate is frozen.

Protocol 10.1 (Post-freeze numerical comparison). *A numerical Paper 12 comparison is admissible if it contains:*

1. the frozen source of α , including certificate hash or external-calibration label;
2. the source of $\hbar, c, m_e, m_p$;
3. the exact unit dictionary;
4. the list of included and excluded corrections;
5. the generated $\mathcal{C}_{H,\text{Bohr}}$ hash;
6. an explicit statement that no hydrogen output was used to modify the α source.

10.2 What may be compared at Paper 12 level

At the gross-spectrum level one may compare:

$$E_n^{(0)}, \quad \nu_{n \rightarrow m}^{(0)}, \quad R_H^{(0)}, \quad a_\mu^{(0)}, \quad (56)$$

with measured quantities only after marking that omitted effects can exceed the desired precision. At modern spectroscopic precision, Paper 12 alone is not enough. Paper 13 corrections are necessary.

10.3 Use of older hydrogen tables

The older hydrogen spectrum notes contain tables comparing Bohr radius, Rydberg constant, ground-state energy, Lamb shift, and fine-structure splitting with experimental values. In the present sequence those tables should be split into two parts:

1. Gross-spectrum entries a_0, R_∞, E_n , allowed in Paper 12 after freeze.
2. Lamb/fine-structure entries, deferred to Paper 13.

They should also carry the label: “computed from post-freeze or externally calibrated α , not used to derive α .”

Figure placeholder: Gross hydrogen energy ladder

Plot or schematic of $E_n = -\mu_H c^2 \alpha^2 / (2n^2)$ for $n = 1, 2, 3, 4$, with Lyman and Balmer transition arrows. Fine and Lamb splittings should be shown only as future correction insets.

Label: `energy-ladder`

Figure 5: Gross hydrogen energy ladder. Plot or schematic of $E_n = -\mu_H c^2 \alpha^2 / (2n^2)$ for $n = 1, 2, 3, 4$, with Lyman and Balmer transition arrows. Fine and Lamb splittings should be shown only as future correction insets.

11 Compatibility with the alpha program

11.1 The full dependency chain

The intended dependency chain is

$$\text{Projection metrology} \longrightarrow \hbar$$

$$\text{Completion cone} \longrightarrow c_\star$$

$$\text{Canonical complexification} \longrightarrow 137$$

(57)

$$\text{Electron holonomy certificate} \longrightarrow \Delta_R \longrightarrow \alpha^{-1} = 137 + \Delta_R$$

$$\text{Mass sector} \longrightarrow m_e, m_p$$

$$\text{Alpha-free hydrogen geometry} \longrightarrow \mathcal{D}_H^{\text{af}}$$

These inputs flow into Paper 12; Paper 12 does not flow backward into them.

11.2 Why hydrogen cannot select the electron holonomy branch

The electron holonomy fraction program requires a branch orientation functional computed before the determinant phase is read. Hydrogen can later test whether the selected branch behaves consistently in neutral-pair systems and antihydrogen, but it cannot select the branch after spectroscopy is known.

Firewall Warning 11.1 (No spectroscopy branch selection). *A branch convention or low-band choice in the electron holonomy certificate is invalid if it is selected because it improves agreement*

with hydrogen spectral lines, Rydberg data, Bohr velocity, Lamb shifts, or fine-structure splittings.

11.3 Hydrogen as a test, not a tuning knob

Once α is frozen, hydrogen becomes a demanding test. If the gross spectrum fails despite correct inputs and corrections, the DTT hydrogen correspondence theorem is wrong. If the gross spectrum succeeds only by changing α or masses line by line, the program fails. If later state-dependent residuals require one free parameter per transition, the differential holonomy program fails.

12 DTT interpretation of the Bohr formulas

12.1 The Bohr radius as projected resonance size

The formula

$$a_\mu = \frac{\hbar}{\mu_H c \alpha} \quad (58)$$

contains the whole DTT constants program in miniature:

- $\hbar$ converts compact temporal phase into record action;
- c converts time-like completion into spatial record scale;
- α supplies the frozen low-energy charged-sector boundary strength;
- μ_H supplies the neutral pair's temporal inertia.

Thus a_μ is not an independent length constant. It is the projected size of a stable neutral temporal resonance.

12.2 Binding energy as phase-lock deficit

The binding energy

$$E_n = -\frac{\mu_H c^2 \alpha^2}{2n^2} \quad (59)$$

can be read as the record-energy deficit associated with phase-locking the electron and proton temporal modes into a stable neutral cycle. The $1/n^2$ scaling is a consequence of Coulomb scale invariance and integer temporal winding.

12.3 Rydberg constant as frequency image

The Rydberg constant is not fundamental in DTT. It is the wavenumber image of the binding scale:

$$R_H = \frac{1}{hc} \frac{\mu_H c^2 \alpha^2}{2}. \quad (60)$$

It exists because the same h and c bridges translate record action/energy into photon frequency and then into wavenumber.

12.4 The electromagnetic tick as a sampling scale

The post-freeze tick $\tau_{\text{em},e} = \alpha\hbar/(m_e c^2)$ is best interpreted as the electromagnetic sampling fraction of the electron reduced Compton recurrence. It should not be treated as a primitive clock that magically explains α . Its explanatory direction is downstream:

$$\alpha \Rightarrow \tau_{\text{em},e}, \text{ not } \tau_{\text{em},e} \Rightarrow \alpha. \quad (61)$$

13 Finite-regulator route from Paper 11 to Paper 12

13.1 Regulated neutral datum

At finite regulator r , Paper 11 supplies $\mathcal{D}_{H,r}^{\text{af}}$. Paper 12 requires a post-freeze activation

$$A_{\alpha,r} : \mathcal{D}_{H,r}^{\text{af}} \rightarrow (\mathcal{H}_{H,r}^{\text{env}}, H_{H,r}^{\text{env}}, Q_{H,r}). \quad (62)$$

The activation should satisfy

$$H_{H,r}^{\text{env}} = K_r^{\text{rel}} - \alpha\hbar c V_r^{\text{Coul}} + \epsilon_r^{\text{corr}}, \quad (63)$$

where $K_r^{\text{rel}} \rightarrow -\nabla^2/(2\mu_H)$, $V_r^{\text{Coul}} \rightarrow 1/r$, and ϵ_r^{corr} records deferred corrections.

13.2 Regulator convergence conditions

Theorem Target 13.1 (Finite-regulator hydrogen spectrum convergence). *There should exist comparison maps $\iota_{r \rightarrow r'}$ between regulated hydrogen envelope spaces such that, for fixed low-lying levels,*

$$|E_{n,r} - E_{n,r'}| \leq A_n \epsilon_{r,r'} + B_n \delta_{r,r'}^{\text{source}}, \quad (64)$$

with $\epsilon_{r,r'} \rightarrow 0$ along the regulator trajectory and $\delta_{r,r'}^{\text{source}} = 0$ if the same post-freeze input object is used.

13.3 Finite-cell resonance isolation

The hydrogen resonance isolation theorem should mirror the low-band isolation theorems in the electron certificate:

$$m_{H,\text{cert}}(r) = \frac{\gamma_*(r)}{M_r}(1 - q_r) - 2C_{\text{loc}}(r)\epsilon_r - \frac{4\ell_r^2}{\Gamma_r} > 0. \quad (65)$$

Here $m_{H,\text{cert}}(r)$ is not an energy level; it is the gap protecting the finite-regulator resonance sector from leakage into non-hydrogenic or continuum sectors.

Finite-regulator status

The continuum Coulomb spectrum is mathematically standard. The DTT finite-regulator theorem is still a target: show that the record complexes and activation maps converge to this envelope without using measured hydrogen spectroscopy to tune the construction.

14 Algorithmic construction protocol

Protocol 14.1 (Paper 12 construction protocol). *The post-freeze hydrogen gross-spectrum construction proceeds as follows.*

1. Import the alpha-free hydrogen datum $\mathcal{D}_H^{\text{af}}$ from Paper 11.
2. Import or declare source statuses for $c_\star, \hbar, \alpha, m_e, m_p$.
3. Verify that α is either an upstream frozen certificate output or explicitly labeled external calibration.
4. Compute $\mu_H = m_e m_p / (m_e + m_p)$.
5. Compute $a_\mu = \hbar / (\mu_H c \alpha)$ and $R_H = \mu_H c \alpha^2 / (2h)$.
6. Compute gross levels $E_n = -\mu_H c^2 \alpha^2 / (2n^2)$ for the declared range.
7. Compute transitions $\nu_{n \rightarrow m} = c R_H (1/m^2 - 1/n^2)$.
8. Generate a correction ledger: reduced mass included; fine, Lamb, hyperfine, nuclear size deferred unless later certificates are supplied.
9. Serialize the certificate canonically and compute h_H .
10. Only after serialization, compare with measured or tabulated hydrogen data.

Listing 2: Pseudocode for Paper 12 gross-spectrum certificate generation.

```

input I_H
assert I_H.alpha.status in {"frozen_dtt_certificate", "external_calibration", "toy"}
assert no_backflow == true
mu = m_e*m_p/(m_e+m_p)
h = 2*pi*hbar
a_mu = hbar/(mu*c*alpha)
R_H = mu*c*alpha**2/(2*h)
for n in levels_requested:
    E[n] = -mu*c**2*alpha**2/(2*n**2)
for (n,m) in transitions_requested:
    nu[n,m] = c*R_H*(1/m**2 - 1/n**2)
certificate = canonical_serialize(inputs, mu, a_mu, R_H, E, nu, ledgers)
hash = sha256(certificate_without_hash)
return certificate_with_hash

```

15 Worked symbolic examples

15.1 Infinite nuclear mass approximation

If $m_p \rightarrow \infty$, then $\mu_H \rightarrow m_e$, and

$$a_\mu \rightarrow a_0 = \frac{\hbar}{m_e c \alpha}, \quad E_n \rightarrow -\frac{m_e c^2 \alpha^2}{2n^2}, \quad R_H \rightarrow R_\infty = \frac{m_e c \alpha^2}{2h}. \quad (66)$$

This is the approximation often used to display the simplest hydrogenic scale relations.

15.2 Finite proton mass

For finite m_p , the leading correction is

$$\frac{\mu_H}{m_e} = \left(1 + \frac{m_e}{m_p}\right)^{-1} = 1 - \frac{m_e}{m_p} + O\left(\frac{m_e^2}{m_p^2}\right). \quad (67)$$

Thus

$$R_H = R_\infty \left(1 - \frac{m_e}{m_p} + O\left(\frac{m_e^2}{m_p^2}\right)\right). \quad (68)$$

The proton–electron mass ratio therefore enters hydrogen spectroscopy already at the gross-spectrum level. A complete DTT derivation of hydrogen numerical spectra ultimately requires a DTT account of m_p/m_e , not only α .

15.3 Toy internal-unit example

Set $\hbar = c = m_e = 1$, $m_p = 100$, and $\alpha = 1/10$. Then

$$\mu_H = \frac{100}{101}, \quad a_\mu = \frac{101}{10}, \quad E_n = -\frac{50}{101} \frac{1}{100n^2} = -\frac{1}{202n^2}. \quad (69)$$

The point of this toy example is not numerical realism. It verifies formula flow and certificate validation in a target-blind setting.

16 Source quarantine and legacy hydrogen formulas

16.1 Legacy formulas that are post-freeze only

The following expressions are allowed in Paper 12 only after source-status freeze:

$$\tau_{\text{em},e} = \alpha \frac{\hbar}{m_e c^2}, \quad (70)$$

$$a_0 = \frac{\hbar}{m_e c \alpha}, \quad (71)$$

$$R_\infty = \frac{m_e c \alpha^2}{2\hbar}, \quad (72)$$

$$E_n = -\frac{m_e c^2 \alpha^2}{2n^2}. \quad (73)$$

They should not appear in pre-freeze alpha-certificate code, hidden comments, parameter names, branch selectors, regulator trajectories, low-band choices, or normalization rules.

Theorem 16.1 (Hydrogen quarantine theorem). *Suppose the dependency graph of a DTT constants derivation satisfies the no-backflow criterion [theorem 2.2](#). Then Paper 12 hydrogen spectral formulas cannot affect the value, branch, regulator trajectory, low-band projector, source normalization, or target comparison of the electron holonomy certificate.*

Proof. By [theorem 2.2](#), the graph contains arrows from upstream constants into $\mathcal{C}_{H,\text{Bohr}}$ but no arrow from $\mathcal{C}_{H,\text{Bohr}}$ to the electron certificate $\mathcal{C}_e$. Any influence of [equations \(70\) to \(73\)](#) on the electron certificate would define such an arrow, contradicting the criterion. Therefore those formulas are quarantined from the pre-freeze alpha derivation. $\square$

16.2 How to cite older hydrogen drafts safely

The older hydrogen drafts may be cited for:

- temporal-resonance intuition;
- inner-time loop diagrams;
- post-freeze derivation of Bohr/Rydberg formulas;
- motivation for radiative temporal-loop corrections.

They should not be cited as a non-circular derivation of α unless the alpha-containing tick formula is independently replaced by a pre-freeze DTT construction.

Citation rule for old hydrogen sources

If a source states that all hydrogen spectroscopy follows from $\Delta t_r = \alpha \hbar / (m_e c^2)$, cite it in Paper 12 as a post-freeze phenomenology source, not as an upstream source for α .

17 Interfaces to Papers 13 and 14

17.1 Preparation for fine structure and Lamb shift

Paper 13 should start from the output of Paper 12 and add correction operators:

$$H_H = H_H^{(0)} + H_{\text{rel}} + H_{\text{spin-orbit}} + H_{\text{Darwin}} + H_{\text{recoil}} + H_{\text{rad}} + H_{\text{finite size}} + \cdots \quad (74)$$

The DTT challenge is to derive each correction from temporal record structure, not to copy coefficients from QED without explanation.

17.2 Preparation for differential holonomy tests

Paper 14 should use the Paper 12 quantum labels (n, ℓ, m) and later (j, F) to define state-dependent residuals

$$\Delta_R(n, \ell, j, F) = \Delta_R(1S) + \eta_{m\ell j F}. \quad (75)$$

The gross spectrum provides the state labels and zeroth-order wavefunctions; it does not determine η by fitting.

17.3 Preparation for mass-ratio tests

Because $\mu_H/m_e = (1 + m_e/m_p)^{-1}$, gross hydrogen already depends on the proton–electron mass ratio. A future DTT mass-ratio paper should provide m_p/m_e internally. Paper 12 can then regenerate isotope shifts and reduced-mass corrections without external mass calibration.

Figure placeholder: Input ledger flow into hydrogen spectrum

A block diagram showing c_* , $\hbar$, α , m_e , m_p ,
and $\mathcal{D}_H^{\text{af}}$ feeding into a_μ , R_H , E_n , and ν_{nm} .

Label: `input-ledger-flow`

Figure 6: Input ledger flow into hydrogen spectrum. A block diagram showing c_* , $\hbar$, α , m_e , m_p , and $\mathcal{D}_H^{\text{af}}$ feeding into a_μ , R_H , E_n , and ν_{nm} .

Figure placeholder: Temporal cycle versus Coulomb envelope

The alpha-free neutral temporal cycle should be shown on the left; the
post-freeze Coulomb envelope and familiar n -level ladder on the right.

Label: `temporal-cycle-vs-envelope`

Figure 7: Temporal cycle versus Coulomb envelope. The alpha-free neutral temporal cycle should be shown on the left; the post-freeze Coulomb envelope and familiar n -level ladder on the right.

Figure placeholder: No-backflow dependency graph

A directed acyclic graph: Papers 1–10 feed $\alpha, \hbar, c, m_e, m_p$ into
Paper 12; arrows back to the electron certificate are forbidden.

Label: `quarantine-graph`

Figure 8: No-backflow dependency graph. A directed acyclic graph: Papers 1–10 feed $\alpha, \hbar, c, m_e, m_p$ into Paper 12; arrows back to the electron certificate are forbidden.

Figure placeholder: Hydrogen correction hierarchy

A scale diagram showing gross α^2 , fine α^4 , Lamb
 $\alpha^5 \ln \alpha^{-1}$, hyperfine, recoil, and finite-size layers.

Label: `correction-hierarchy`

Figure 9: Hydrogen correction hierarchy. A scale diagram showing gross α^2 , fine α^4 , Lamb $\alpha^5 \ln \alpha^{-1}$, hyperfine, recoil, and finite-size layers.

18 Figures and diagrams planned for a publication version

19 Envelope wavefunctions and record-density interpretation

19.1 Why wavefunctions appear in a record theory

DTT does not treat the wavefunction as a primitive object living in an independently given spatial container. In the post-freeze hydrogen envelope it is a compact way of encoding stable record-amplitude distributions after the neutral temporal resonance has been projected into a continuum comparison layer. Thus the ordinary hydrogen wavefunctions are not rejected; they are reinterpreted as envelope descriptions of projected record densities.

Definition 19.1 (Record-envelope wavefunction). *A record-envelope wavefunction for hydrogen is a continuum representative $\psi_{n\ell m}$ of a stable finite-regulator family of projected hydrogen record amplitudes, normalized so that*

$$\int_{\mathbb{R}^3} |\psi_{n\ell m}(x)|^2 d^3x = 1. \quad (76)$$

It is an envelope observable, not a pre-projection ontology.

The standard bound-state functions take the separated form

$$\psi_{n\ell m}(r, \theta, \phi) = R_{n\ell}(r) Y_{\ell m}(\theta, \phi), \quad (77)$$

where $Y_{\ell m}$ are spherical harmonics and

$$R_{n\ell}(r) = \mathcal{N}_{n\ell} \left(\frac{2r}{na_\mu} \right)^\ell e^{-r/(na_\mu)} L_{n-\ell-1}^{2\ell+1} \left(\frac{2r}{na_\mu} \right). \quad (78)$$

Here L_k^a denotes an associated Laguerre polynomial and $\mathcal{N}_{n\ell}$ is the normalization constant.

19.2 Radial expectation values as projected sizes

The leading expectation values are useful because they translate temporal resonance labels into projected sizes. For example,

$$\langle r \rangle_{n\ell} = \frac{a_\mu}{2} (3n^2 - \ell(\ell+1)), \quad (79)$$

and

$$\langle r^{-1} \rangle_{n\ell} = \frac{1}{a_\mu n^2}. \quad (80)$$

In the DTT interpretation, these are not primitive orbital radii. They are statistics of the projected record envelope. The Bohr radius a_μ is the universal length scale controlling those statistics once the constants are frozen.

19.3 Degeneracy and hidden symmetry

The Coulomb Hamiltonian has an enlarged symmetry responsible for the n^2 orbital degeneracy before spin. In ordinary language this is related to the Runge–Lenz vector. In DTT language it

is a sign that the leading post-freeze envelope retains more symmetry than the full finite record structure. Corrections in Paper 13 break this symmetry; the differential holonomy program in Paper 14 may test whether the breaking follows DTT record-cycle patterns rather than arbitrary line-by-line adjustment.

Theorem Target 19.2 (Record origin of Coulomb hidden symmetry). *A complete DTT hydrogen theory should explain the leading Coulomb degeneracy as a symmetry of the post-freeze neutral-pair record envelope and should identify which finite-regulator features break it into fine, Lamb, hyperfine, and holonomy-residual substructures.*

**Figure placeholder: Record-envelope
density for low hydrogen states**

A future figure should show $|\psi_{100}|^2$, $|\psi_{200}|^2$, and $|\psi_{21m}|^2$ as projected record-density envelopes, not literal point-particle trajectories.

Label: `record-envelope-density`

Figure 10: Record-envelope density for low hydrogen states. A future figure should show $|\psi_{100}|^2$, $|\psi_{200}|^2$, and $|\psi_{21m}|^2$ as projected record-density envelopes, not literal point-particle trajectories.

20 Isotopes, positronium, muonium, and hydrogen-like systems

20.1 Why extensions matter

A theory that only reproduces ordinary hydrogen after receiving α, m_e, m_p as inputs is not yet deeply tested. The same post-freeze formulas should also organize deuterium, muonium, positronium, and hydrogen-like ions. These systems separate the roles of reduced mass, nuclear charge, finite nuclear structure, and annihilation/radiative sectors.

20.2 Deuterium and isotope shifts

For deuterium, replace m_p by the deuteron mass m_d :

$$\mu_D = \frac{m_e m_d}{m_e + m_d}, \quad R_D = R_\infty \frac{\mu_D}{m_e}. \quad (81)$$

At Paper 12 level, the isotope shift is primarily a reduced-mass effect:

$$\nu_{n \rightarrow m}^D - \nu_{n \rightarrow m}^H = c R_\infty \left(\frac{\mu_D - \mu_H}{m_e} \right) \left(\frac{1}{m^2} - \frac{1}{n^2} \right) + \dots \quad (82)$$

The ellipsis includes nuclear size and polarizability corrections, which are not Paper 12 results.

20.3 Positronium

For positronium, m_p is replaced by another electron mass and $\mu = m_e/2$. The gross spectrum is therefore

$$E_n^{\text{Ps}} = -\frac{m_e c^2 \alpha^2}{4n^2}, \quad (83)$$

half the infinite-proton hydrogen binding scale. Positronium also has annihilation and spin structure that cannot be captured by Paper 12 alone. It is nevertheless an important test of whether the mass and charge-conjugation components of DTT are internally consistent.

20.4 Muonium

Muonium replaces the proton with an antimuon. Its gross spectrum uses

$$\mu_{\text{Mu}} = \frac{m_e m_\mu}{m_e + m_\mu}. \quad (84)$$

Because the nucleus-like constituent is pointlike to a much better approximation than the proton, muonium is an excellent future test for separating proton-structure effects from pure temporal-mass and charge-sector effects.

20.5 Hydrogen-like ions

For a nucleus of charge Z , the leading spectrum is

$$E_n^{Z,N} = -\frac{\mu_N c^2 (Z\alpha)^2}{2n^2}, \quad a_{\mu,Z} = \frac{\hbar}{\mu_N c Z \alpha}. \quad (85)$$

As $Z\alpha$ grows, relativistic and QED corrections become increasingly important. Therefore Paper 12 should be used only as the leading term in a controlled hierarchy.

System	Paper 12 input change	Additional later corrections
Hydrogen	$M_N = m_p, Z = 1$	proton radius, hyperfine, Lamb shift.
Deuterium	$M_N = m_d, Z = 1$	deuteron size, polarizability, nuclear spin.
Positronium	$\mu = m_e/2$	annihilation, equal-mass recoil, spin states.
Muonium	$M_N = m_\mu, Z = 1$	muon magnetic moment, weak decay, recoil.
Hydrogen-like ion	$Z > 1$	high- Z relativistic/QED and nuclear effects.

21 Numerical reporting template

21.1 Why numbers require labels

A numerical table is useful only if the source of each input is declared. The same formula can mean three different things depending on status:

1. *DTT-derived*: all constants come from upstream DTT certificates.
2. *externally calibrated*: one or more constants are imported from a standards body or experiment.
3. *toy benchmark*: values are nonphysical and used only to test formula flow.

Only the first status would support a strong derivation claim. The second supports a consistency or phenomenology check. The third supports software/protocol validation only.

21.2 Recommended table columns

Quantity	Formula	Source status	Comment
μ_H	$m_e m_p / (m_e + m_p)$	mass input	Needed even at gross-spectrum level.
a_μ	$\hbar / (\mu_H c \alpha)$	post-freeze	Not an input to α .
R_H	$\mu_H c \alpha^2 / (2h)$	post-freeze	Compare only after freeze.
E_1	$-\mu_H c^2 \alpha^2 / 2$	post-freeze	Gross binding only.
$\nu_{2 \rightarrow 1}$	$c R_H (1 - 1/4)$	post-freeze	Requires corrections for precision.
$\tau_{\text{em},e}$	$\alpha \hbar / (m_e c^2)$	post-freeze	Derived sampling tick, not primitive.

21.3 Precision warning

At present-day spectroscopy precision, gross-spectrum formulas are not sufficient. A table that compares E_n or R_H against high-precision values must state which corrections are omitted. A Paper 12 comparison can be pedagogically useful, but a high-precision physical comparison belongs to Papers 13 and 14.

22 Assumption registry and failure modes

22.1 Assumptions

ID	Assumption	Failure consequence
A1	$\mathcal{I}_H$ is frozen and source-labeled.	Numerical comparison is unauditale.
A2	α is not taken from hydrogen spectroscopy unless labeled external calibration.	Circular derivation of α .
A3	Neutral-to-Coulomb correspondence holds at leading order.	DTT does not explain why hydrogen obeys the Coulomb envelope.
A4	Reduced-mass approximation captures leading two-body effect.	Gross-spectrum formulas need a different leading kinetic sector.
A5	Record-envelope continuum limit is valid for low states.	Finite-regulator construction must replace continuum formulas.
A6	Correction ledger is honest.	Overclaiming fine/Lamb/hyperfine precision from Paper 12.

22.2 Failure modes

The strong Paper 12 program fails or weakens if:

1. the neutral-to-Coulomb correspondence cannot be derived from DTT record geometry;
2. the same frozen α that enters the electron boundary response does not enter hydrogen;
3. hydrogen spectroscopy is used to alter the electron holonomy certificate;
4. transition-by-transition fitting is required at the gross-spectrum level;
5. mass-ratio inputs remain empirical while the paper claims a complete derivation of hydrogen numerics;
6. radiative corrections are imported from QED while claimed as DTT derivations.

23 Conclusion

This paper completes the safe transition from alpha-free hydrogen geometry to post-freeze hydrogen spectroscopy. The result is intentionally modest but essential. Once the upstream constants c_* , $\hbar$, α , m_e , and m_p are frozen or calibrated, the gross hydrogen spectrum follows:

$$a_\mu = \frac{\hbar}{\mu_H c \alpha}, \quad E_n = -\frac{\mu_H c^2 \alpha^2}{2n^2}, \quad R_H = \frac{\mu_H c \alpha^2}{2h}. \quad (86)$$

The inner-time electromagnetic tick becomes

$$\tau_{\text{em},e} = \alpha \frac{\hbar}{m_e c^2}, \quad (87)$$

but only as a derived post-freeze scale.

The key scientific boundary is unchanged:

Paper 12 uses α to reconstruct hydrogen; it does not use hydrogen to derive α .

The next paper should add the correction hierarchy: relativistic fine structure, spin-orbit and Darwin terms, Lamb shift, recoil, finite nuclear size, hyperfine structure, and temporal-loop radiative coefficients. Paper 14 should then use the resulting state labels for differential holonomy tests.

A Appendix A: Derivation details for the Bohr formulas

Starting from

$$\mu v^2 / r = \alpha \hbar c / r^2, \quad \mu v r = n \hbar, \quad (88)$$

we get $r = n \hbar / (\mu v)$. Substitute into the force equation:

$$\mu v^2 \frac{\mu v}{n \hbar} = \alpha \hbar c \frac{\mu^2 v^2}{n^2 \hbar^2}. \quad (89)$$

Canceling $\mu^2 v^2 / \hbar$ yields $v = \alpha c / n$. Then

$$r_n = \frac{n^2 \hbar}{\mu c \alpha}, \quad (90)$$

and

$$E_n = \frac{1}{2}\mu v^2 - \frac{\alpha \hbar c}{r_n} = \frac{\mu c^2 \alpha^2}{2n^2} - \frac{\mu c^2 \alpha^2}{n^2} = -\frac{\mu c^2 \alpha^2}{2n^2}. \quad (91)$$

B Appendix B: Source-role ledger

Source class	Allowed role	Forbidden role
Projection metrology papers	Provide unit dictionary, structural bridge logic, $\hbar$ role, $c_\star$ role.	Set α from hydrogen data.
Canonical complexification paper	Provide structural integer 137.	Supply fractional residual or Bohr spectrum.
Holonomy fraction/certificate papers	Supply or target α .	Use hydrogen spectroscopy pre-freeze to choose branch or low band.
Paper 11 alpha-free hydrogen	Provide temporal neutral-pair geometry, phase opposition, orientation interface.	Use Bohr/Rydberg formulas as alpha-free inputs.
Legacy hydrogen tick papers	Provide post-freeze intuition and diagrams.	Derive α by inserting α into Δt_r .
Experimental spectroscopy	Post-freeze comparison only.	Tune upstream constants or select certificate branch.

C Appendix C: Machine-readable schema sketch

The package includes a JSON schema and validator. The core schema requires fields for constants, source statuses, no-backflow, correction ledger, levels, transitions, and canonical hash. The schema is intentionally strict about the source status of α : it must be `frozen_dtt_certificate`, `external_calibration`, or `toy_post_freeze`; it cannot be `from_hydrogen_fit`.

Listing 3: Schema sketch for the alpha source-status check.

```
"alpha": {
  "value": "number or symbolic",
  "status": ["frozen_dtt_certificate", "external_calibration", "toy_post_freeze"],
  "forbidden_status": ["from_hydrogen_fit", "from_rydberg_fit", "from_lamb_shift_fit"]
}
```

D Appendix D: Referee stress tests

1. **Is this just standard hydrogen?** The gross-spectrum algebra is standard. The DTT content is the source-status discipline, temporal-record interpretation, and neutral-to-Coulomb correspondence target.

2. **Did the paper derive α ?** No. It uses α post-freeze. The derivation target for α belongs to Papers 7–10.
3. **Is the old tick formula circular?** It is circular if used before α is fixed. This paper uses it only after freeze as $\tau_{\text{em},e} = \alpha\hbar/(m_e c^2)$.
4. **Does the paper predict the Lamb shift?** No. It prepares the hierarchy and defers radiative temporal-loop coefficients to Paper 13.
5. **Can the paper be numerically tested?** Yes, after input sources and corrections are declared. Gross-spectrum comparisons alone must not be advertised as precision QED-level tests.
6. **Can a failed neutral-to-Coulomb correspondence falsify the DTT hydrogen program?** Yes. If DTT cannot derive the Coulomb envelope from the alpha-free neutral record geometry plus frozen charged response, the hydrogen interpretation remains incomplete.

E Appendix E: Notation ledger

Symbol	Meaning
$c_\star$	DTT local-completion speed / causal-cone invariant.
$\hbar$	action-phase bridge constant; $h = 2\pi\hbar$.
α	low-energy on-shell fine-structure constant after freeze.
m_e, m_p	electron and proton rest-mass inputs or later DTT mass-sector outputs.
μ_H	reduced hydrogen mass $m_e m_p / (m_e + m_p)$.
a_μ	reduced Bohr radius $\hbar / (\mu_H c \alpha)$.
R_H	reduced-mass Rydberg wavenumber $\mu_H c \alpha^2 / (2h)$.
E_n	leading gross Coulomb energy $-\mu_H c^2 \alpha^2 / (2n^2)$.
$\tau_{\text{em},e}$	post-freeze electromagnetic sampling tick $\alpha\hbar / (m_e c^2)$.
$\mathcal{D}_H^{\text{af}}$	alpha-free neutral hydrogen record datum from Paper 11.
$\mathcal{C}_{H,\text{Bohr}}$	post-freeze gross-spectrum certificate.

References

- [1] M. H. Yousef, *Dual-Time Projection Realism: Generative Order, Stable Records, and Observable Time*, DTT Constants and Hydrogen Series, Paper 1, expanded draft, 2026.
- [2] M. H. Yousef, *Projection Metrology and Constants*, DTT Constants and Hydrogen Series, Paper 2, expanded draft, 2026.
- [3] M. H. Yousef, *The Structural Emergence of $\hbar$* , DTT Constants and Hydrogen Series, Paper 3, expanded draft, 2026.
- [4] M. H. Yousef, *Emergent Additive Record Action from Dual-Time Projection and Stabilization*, DTT Constants and Hydrogen Series, Paper 4, expanded draft, 2026.
- [5] M. H. Yousef, *Local Completion Cones and $c_\star$* , DTT Constants and Hydrogen Series, Paper 6, expanded draft, 2026.

- [6] M. H. Yousef, *Canonical Complexification and the Structural Integer 137*, DTT Constants and Hydrogen Series, Paper 7, expanded draft, 2026.
- [7] M. H. Yousef, *The Holonomy Fraction in Duality of Time Theory*, DTT Constants and Hydrogen Series, Paper 8, expanded draft, 2026.
- [8] M. H. Yousef, *The Coupling-Correspondence Theorem*, DTT Constants and Hydrogen Series, Paper 9, expanded draft, 2026.
- [9] M. H. Yousef, *The Physical Electron Certificate*, DTT Constants and Hydrogen Series, Paper 10, expanded draft, 2026.
- [10] M. H. Yousef, *Temporal Reconstruction of Hydrogen I: Alpha-Free Proton–Electron Record Geometry*, DTT Constants and Hydrogen Series, Paper 11, expanded draft, 2026.
- [11] N. Bohr, *On the Constitution of Atoms and Molecules*, Philosophical Magazine, 1913.
- [12] E. Schroedinger, *Quantisierung als Eigenwertproblem*, Annalen der Physik, 1926.
- [13] M. H. Yousef, *Temporal Reconstruction of the Hydrogen Atom: A Spectral Model from the Duality of Time Theory*, DTT/SMM draft, 2025.
- [14] M. H. Yousef, *Inner-Time Ticks and the Hydrogen Atom: A DTT Explanation of Energy Levels and Spectrum*, DTT draft, 2025.